J. Natn. Sci. Coun. Sri Lanka 1981 9 (2) : 153—156

An Extension of a Test of Equality Between Sets of Coefficients in Two Linear Regressions with Unequal Disturbance Variances to Accommodate more than Two Regressions

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(*Date of receipt* : 28 *July* 1980)
(*Date of acceptance* : 06 *May* 1981)

Abstract : A test for testing the equality of coefficients of linear regression models is proposed.

1. Introduction

The author considered the problem of testing equality between sets of coefficients in two linear regressions when disturbance variances are unequal and proposed an exact test using generalised T^2 statistic.[3] The main idea of the present article is to generalise the above result to accommodate more than two regressions.

2. Result

Consider the models

$$Y_i = X_i \beta_i + \epsilon_i, \ \epsilon_i \sim N(O, \sigma_i^2 I_{n_i}) \qquad (1)$$
$$E\, \epsilon_i \epsilon_j = 0 \text{ for } i \neq j,\ i,j = 1, 2, \ldots, p$$

where Y_i and X_i are $n_i \times 1$ and $n_i \times k$ observation matrices, β_i is a $k \times 1$ coefficient matrix, and ϵ_i is an $n_i \times 1$ disturbance matrix for i = 1, 2, ..., p.

Assume all X_i have full column rank, $n_i > k$ for i = 1, 2, ..., p. Combining all p models together we have

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_p \end{bmatrix} = \begin{bmatrix} X_1 & & & \\ & X_2 & & \\ & & \ddots & \\ & & & X_p \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_p \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_p \end{bmatrix}$$

or $Y = X\beta + \epsilon$ with $\epsilon \sim N(O, \Sigma)$ (2)

where

$$\Sigma = \begin{bmatrix} \sigma_1^2 I_{n_1} & & & \\ & \sigma_2^2 I_{n_2} & & \\ & & \ddots & \\ & & & \sigma_p^2 I_{n_p} \end{bmatrix}$$

Then the ordinary least squares estimator of β is given by

$$\hat{\beta} = (X^1X)^{-1} X^1Y = \beta + (X^1X)^{-1} X^1\epsilon$$

and $\hat{\beta} \sim N(\beta, \Omega)$ (3)

where

$$\Omega = \begin{bmatrix} \sigma_1^2 (X_1^1X_1)^{-1} & & & O \\ & \sigma_2^2 (X_2^1X_2)^{-1} & & \\ & & \ddots & \\ O & & & \sigma_p^2 (X_p^1X_p)^{-1} \end{bmatrix}$$

Assume we want to test the restrictions $R\beta = r$, where R is $mxpk$ matrix of restrictions and r is a mxl matrix of constants, on the parameters of the model (2).

From (3) we have $R\hat{\beta} \sim N(R\beta, R\Omega R^1)$.

Under the null hypothesis $H_0 : R\beta = r$

$$R\hat{\beta} \sim N(r, R\Omega R^1), \tag{4}$$

Let $M_i = I_{n_i} - X_i (X_i^1 X_i)^{-1} X_i^1 = Z_i Z_i^1$

Where $Z_i^1X_i = O$ and $Z_i^1Z_i = I_{n_i} - k$. It may be noted that the columns of Z_i are eigen vectors of M_i corresponding to unit roots. Now consider $\epsilon_i^* = Z_i^1\epsilon_i$ and note that

$$\epsilon_i^* \sim N(O, \sigma_i^2 I_{n_i} - k) \text{ for } i = 1, 2, \ldots\ldots\ldots, p.$$

Let q be the largest integer less than or equal to

$$\text{minimum} \left\{ \frac{n_1 - k}{k}, \frac{n_2 - k}{k}, \ldots\ldots, \frac{n_p - k}{k} \right\}.$$

If $q \geqslant m$, we can proceed as follows.

Partition each ϵ_i into q sub vectors $\epsilon_i{}^{\star}{}_{(1)}, \epsilon_i{}^{\star}{}_{(2)}, \ldots, \epsilon_i{}^{\star}{}_{(q)}$, each sub vector having k elements.

Let

$$\epsilon_{(j)}{}^{\star} = \begin{bmatrix} \epsilon_1{}^{\star}{}_{(j)} \\ \epsilon_2{}^{\star}{}_{(j)} \\ \cdot \\ \cdot \\ \epsilon_p{}^{\star}{}_{(j)} \end{bmatrix}$$

and let Q_i be krk matrix such that $Q_i^1 \, Q_i = (X_i^1 X_i)^{-1}$.

Define

$$Q = \begin{bmatrix} Q_1 & & & & \\ & Q_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & Q_p \end{bmatrix}$$

Then define $\eta_j \sim RQ^1 \epsilon^{\star}{}_{(j)} \qquad j = 1, 2, \ldots\ldots\ldots, q.$ (5)

Therefore we have $\eta_j \sim N\,(O,\ R\Omega R^1) \quad j = 1, 2, \ldots\ldots\, q.$ (6)

From (3) we also have $R\hat{\beta} \sim N\,(r,\ R\Omega R^1)$
under $H_o : R\beta = r$.

Then from T. W. Anderson[1] it follows that

$$\frac{(R\hat{\beta})^1 \; S^{-1} \; (R\hat{\beta})}{q} \cdot \frac{q - m + 1}{m} \qquad (7)$$

is distributed as non-central F distribution with m and $q - m + 1$ degrees of freedom and non-centrality parameter $r^1\,(R\Omega R^1)^{-1}\, r$.

where $S = \dfrac{1}{q} \sum\limits_{j=1}^{q} \eta_j \, \eta_j^1$.

Under the null hypothesis $R\beta = O$ the statistic (7) is distributed as central F with corresponding degrees of freedom.

3. A special case

If one wants to test the hypothesis of equality among the entire sets of coefficients of all the regressions he should formulate the restrictions as follows :

$$\begin{bmatrix} I-I\ O & \cdots & O\ O \\ O\ I-I & \cdots & O\ O \\ \vdots & \ddots & \vdots \\ O\ O\ O & \cdots & I-I \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{bmatrix} = \begin{bmatrix} O \\ O \\ \vdots \\ O \end{bmatrix}$$

or $\bar{R}\beta = O$ (8)

where all unit and zero matrices in the equation (8) are of order $k \times k$

For this particular case $m = k\ (p-1)$. Hence the test statistic (7) becomes

$$\frac{(\bar{R}\hat{\beta})^1\ S^{-1}\ (\bar{R}\hat{\beta})}{q} \cdot \frac{q-k\ (p-1)+1}{k\ (p-1)} \quad (9)$$

This statistic is distributed as central F with $k\ (p-1)$ and $q-k\ (p-1)+1$ degrees of freedom provided $q \geqslant k\ (p-1)$. Relating this result to the case of two regressions where $p=2$ the statistic (9) reduces to

$$\frac{(\hat{\beta}_1-\hat{\beta}_2)^1\ S^{-1}\ (\hat{\beta}_1-\hat{\beta}_2)}{q} \cdot \frac{q-k+1}{k}$$

which is the test suggested by the author.[3]

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