J. Natn. Sci. Coun. Sri Lanka 1979 7(2) : 75—77

Internal Magnetic Fields of White Dwarf Stars : Circulation Effects

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(*Paper accepted :* 03 *July* 1979)

Abstract : It is shown that in the case of white dwarf stars, circulation effects are unlikely to prevent strong internal magnetic fields from being observed.

1. Introduction

The existence of strong magnetic fields ($\gtrsim 10^{11}$ gauss) in the interiors of white dwarf stars can affect their structure and hence their theoretical mass-radius relationship.[7] Most white dwarfs have no observed magnetic field ; the upper limit for the average surface field being of the order of 10^5 gauss.[8] A fraction ($\sim 10^{-2}$) of white dwarfs do have strong (average) surface fields $\sim 10^6$ to 10^8 gauss.[15] In this work, however, we shall be concerned with white dwarfs with no observable fields and, in particular, determine whether such stars can have strong magnetic fields in their interiors. Chanmugam and Gabriel[3] have discussed this problem for non-poloidal rotating white dwarfs with no internal motions. The time scale of decay for magnetic fields $\tau_n \sim \tau_0/(n+1)^2$ where $\tau_0 \sim 5 \times 10^9$ yr and n is the order of the mode considered. Hence, few modes survive over the typical age of a white dwarf $\gtrsim 10^8$ yr. Since the magnetic field would have to be a linear superposition of the first few modes, such a field would, in general, be unlikely to be both strong in the interior and weak at the surface. However, as pointed out in that paper, the presence of meridional circulation could affect the arguments given. Recently, Kippenhahn and Möllenhoff[6] have discussed the possible existence of circulation in rotating white dwarfs. Similarly, we may envisage the existence of circulation in white dwarfs which contain strong magnetic fields. The purpose of this paper is to consider the effects of circulation on the magnetic field (and vice versa) and to determine whether circulation can prevent a strong internal field from being observed.

2. Circulation Effects

We consider three possible cases for the magnetic field of the white dwarf: (*a*) that it was formed with a strong surface field which is then confined beneath the surface by circulation, (*b*) that it was formed with a weak fossil field which was internally amplified by the dynamo action of circulations, (*c*) that it was formed with a strong fossil field lying entirely beneath the surface.

Consider case (*a*). We first suppose that the circulation is driven by stellar rotation with angular velocity Ω. We denote the ratio of the centrifugal force to the gravitational force by $\lambda_r \sim \Omega^2 R^3/GM$, where M and R are the mass and radius of the white dwarf and G the gravitational constant. Let us denote the ratio of the magnetic force to the gravitational force by $\lambda_m = \bar{B}^2 R^4/GM^2$ where $\bar{B}$ is the mean field. For most white dwarfs $\Omega \lesssim 0.1$ radians s^{-1},[4] so that, taking $R \sim 10^9$ cm, $M \sim M_\odot$, we have $\lambda_r \lesssim 0.1$. We note that even if $\lambda_r \sim 1$ the Eddington-Vogt circulation time τ_{EV} is roughly of the same order or slightly longer than the cooling time τ_{COOL} of white dwarfs.[6] Since $\tau_{COOL} \sim 10^9$ yr, the circulation does not have sufficient time to turn over an adequate number of times during the lifetime of the white dwarf.

Secondly, we consider the possibility that $\lambda_m \geqslant \lambda_r$ so that the circulation is driven by the magnetic field itself. In a rotating magnetic star, the poloidal component of the equation of mechanical equilibrium is:

$$\nabla p = \rho \nabla \phi + \rho \Omega^2 \varpi + (\,\mathrm{curl}\,\mathbf{B}\,) \wedge \mathbf{B}/4\pi \qquad (1)$$

where p is the pressure, ϕ the gravitational potential and cylindrical coordinates (ϖ, θ, z) are used. The general solution of the complete set of equations, magnetohydrodynamic and hydrostatic, of which equation (1) is one, is of formidable difficulty. However, an estimate of the magnetic effects may be made as follows: we note that the ratio of the centrifugal force term to the magnetic force term in equation (1) is roughly of order $\rho\Omega^2 R/(\bar{B}^2/4\pi R) \simeq 3M\Omega^2/\bar{B}^2 R \sim \lambda_r/\lambda_m$. Hence, when $\lambda_r \leqslant \lambda_m$ and $\lambda_m \sim 1$ one should obtain the same order of magnitude for the circulation speed as in the case when $\lambda_m \leqslant \lambda_r$ and $\lambda_r \sim 1$. Thus a strong magnetic field cannot cause adequate circulation to choke off the field. We note here that λ_m cannot be larger than one, as then the star would be unstable because its magnetic energy would be greater in magnitude than its gravitational energy.[2] Clearly, also the case $\lambda_r \approx \lambda_m \sim 1$ would not significantly change the speed of circulation to affect the above discussion.

We now turn to case (*b*). From the preceding discussion, we note that the largest possible values for the speed of the circulation, whether driven by rotation or by magnetic fields, are insufficient to produce large poloidal fields by dynamo action.

We thus discuss the possibility that differential rotation creates a strong magnetic field which necessarily must be toroidal. If we choose the mean rotational velocity $v_{rot} \sim 0.1\ R \sim 10^8$ cm s^{-1} the strength of the toroidal field generated $\sim (4\pi\rho\ v^2_{rot})^{\frac{1}{2}} \sim 10^{11}$ gauss. Such a strong toroidal field is unstable in the absence of an equally strong poloidal field.[9,10]

Finally, we consider case (*c*) where the white dwarf is formed with a strong internal field but weak at the surface. Since the effects of circulation, whether driven by rotation or magnetism, are unimportant, this case essentially reduces to the case of a static white dwarf star discussed in the introduction and in the paper by Chanmugam and Gabriel.[3] Such a field would diffuse out of the star on a time scale, at most, of order 10^8 yr, which is less than the typical age of most white dwarfs.

In conclusion, we point out that it is difficult to justify the existence of magnetic fields of sufficient strength which can alter the structure of the star and yet not appear above the surface.

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