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RESEARCH ARTICLE

Estimation of general parameter in adaptive cluster sampling using two auxiliary variables

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Abstract: In this article, exponential ratio type estimators are proposed for general parameter in adaptive cluster sampling. The estimators utilise information on two auxiliary variables in three different situations, i.e. partial, no and full information about population parameters of auxiliary variables. The proposed estimators for general parameter can be used to estimate population mean, coefficient of variation, standard deviation and variance of the variable of interest. The bias and mean square error equations for the proposed estimators are derived using first order approximation. The proposed estimators are more efficient than usual sample estimators and ratio estimators in all three situations under adaptive cluster sampling. Two different populations are used for numerical illustration.

Keywords: Adaptive cluster sampling, auxiliary variable, coefficient of variation.

INTRODUCTION

In the theory of survey sampling, estimation of different population parameters is widely considered using several sampling designs. Conventional sampling designs may not be helpful when the population under consideration is rare and clustered. Thompson (1990) introduced adaptive cluster sampling for such situations as an alternative. The scheme provides higher sampling yields for highly clumped, sporadic and hidden populations.

Adaptive cluster sampling is valuable for studies on drug addicts, endangered species of animals, contagious diseases, rare and precious plants, minerals and natural resources. In adaptive cluster sampling, an initial sample is selected by some conventional sampling design. All the selected units are examined for a predefined condition C. If any of these units satisfy the condition, its neighbouring units are added to the sample. Further, if any of these newly added units satisfy the condition, its neighbouring units are also included in the sample and the process continues until no more units satisfy the condition. Estimators for general parameter under three different situations are proposed.

1. When population mean and variance are known for one auxiliary variable (say z) and unknown for the other auxiliary variable (say x).
2. When population mean and variance are unknown for both auxiliary variables.
3. When population mean and variance are known for both auxiliary variables.

When population parameters for at least one of the auxiliary variable are unknown, we adopt the two phase sampling scheme using adaptive cluster sampling at each phase as follows:

1. In phase one, a large sample of size n' is drawn using adaptive cluster sampling based on auxiliary variable and unknown parameters of auxiliary variables (x and z) are estimated.

2. In phase two, network structure of the phase one sample is used to select a sample of size n $(n < n')$ using adaptive cluster sampling based on the study

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variable. Thus information on the study variable (y) and auxiliary variables (x and z) is observed.

Consider a finite population of N units with labels $1,2,...,N$. Let y_i and (x_i, z_i) be the observed values of the study and the auxiliary variables and w_{y_i} and (w_{x_i}, w_{z_i}) be the network level means. Also let $\bar{w}_y$ and ($\bar{w}_x, \bar{w}_z$) be the sample means and $s^2_{w_y}$ and ($s^2_{w_x}, s^2_{w_z}$) be the sample variances of study and auxiliary variables. Let $\bar{Y}_w$ and ($\bar{X}_w, \bar{Z}_w$) and $S^2_{w_y}$ and ($S^2_{w_x}, S^2_{w_z}$) be the corresponding population means and variances respectively, in transformed population.

The following notations are used:

$$\bar{w}_y = \frac{1}{n}\sum_{i=1}^{n} w_{y_i}, \bar{w}_x = \frac{1}{n}\sum_{i=1}^{n} w_{x_i}, \bar{w}_z = \frac{1}{n}\sum_{i=1}^{n} w_{z_i},$$

$$w_{x_i} = \frac{1}{m_i}\sum_{j\in\Psi_i} x_j, w_{z_i} = \frac{1}{m_i}\sum_{j\in\Psi_i} z_j,$$

$$\bar{Y}_w = \frac{1}{N}\sum_{i=1}^{N} w_{y_i}, \bar{X}_w = \frac{1}{N}\sum_{i=1}^{N} w_{x_i}, \bar{Z}_w = \frac{1}{N}\sum_{i=1}^{N} w_{z_i},$$

$$\bar{w}'_x = \frac{1}{n'}\sum_{i=1}^{n'} w_{x_i}, \bar{w}'_z = \frac{1}{n'}\sum_{i=1}^{n'} w_{z_i},$$

$$s^2_{w_y} = \frac{1}{n-1}\sum_{i=1}^{n}(w_{y_i} - \bar{w}_y)^2, s^2_{w_x} = \frac{1}{n-1}\sum_{i=1}^{n}(w_{x_i} - \bar{w}_x)^2,$$

$$s^2_{w_z} = \frac{1}{n-1}\sum_{i=1}^{n}(w_{z_i} - \bar{w}_z)^2,$$

$$s^{2'}_{w_x} = \frac{1}{n'-1}\sum_{i=1}^{n'}(w_{x_i} - \bar{w}'_x)^2, s^{2'}_{w_z} = \frac{1}{n'-1}\sum_{i=1}^{n'}(w_{z_i} - \bar{w}'_z)^2,$$

$$S^2_{w_y} = \frac{1}{N-1}\sum_{i=1}^{N}(w_{y_i} - \bar{Y}_w)^2,$$

$$S^2_{w_x} = \frac{1}{N-1}\sum_{i=1}^{N}(w_{x_i} - \bar{X}_w)^2, S^2_{w_z} = \frac{1}{N-1}\sum_{i=1}^{N}(w_{z_i} - \bar{Z}_w)^2,$$

$$C_{w_y} = \frac{S_{w_y}}{\bar{Y}_w}, C_{w_x} = \frac{S_{w_x}}{\bar{X}_w},$$

where,

$$C_{w_z} = \frac{S_{w_z}}{\bar{Z}_w}, C_{w_y w_x} = \rho_{w_y w_x} C_{w_y} C_{w_x}, C_{w_x w_z} =$$

$$\rho_{w_x w_z} C_{w_x} C_{w_z}, C_{w_y w_z} = \rho_{w_y w_z} C_{w_y} C_{w_z}.$$

The following error terms are used:

$$e_0 = \frac{\bar{w}_y - \bar{Y}_w}{\bar{Y}_w}, e_1 = \frac{\bar{w}_x - \bar{X}_w}{\bar{X}_w}, e'_1 = \frac{\bar{w}'_x - \bar{X}_w}{\bar{X}_w},$$

$$e_2 = \frac{\bar{w}_z - \bar{Z}_w}{\bar{Z}_w}, e'_2 = \frac{\bar{w}'_z - \bar{Z}_w}{\bar{Z}_w},$$

$$e_3 = \frac{s^2_{w_y} - S^2_{w_y}}{S^2_{w_y}}, e_4 = \frac{s^2_{w_x} - S^2_{w_x}}{S^2_{w_x}}, e'_4 = \frac{s^{2'}_{w_x} - S^2_{w_x}}{S^2_{w_x}},$$

$$e_5 = \frac{s^2_{w_z} - S^2_{w_z}}{S^2_{w_z}}, e'_5 = \frac{s^{2'}_{w_z} - S^2_{w_z}}{S^2_{w_z}},$$

such that $E(e_i) = 0 \quad \forall i = 0,1,2,3,4,5$ and $E(e'_i) = 0 \, \forall i = 1,2,4,5.$

Also,

$$E(e_0^2) = \theta C^2_{w_y}, E(e_2 e_3) = \theta\lambda_{201} C_{w_z}, E(e'_1 e'_2) = E(e_1 e'_2)$$

$$= E(e_2 e'_1) = \theta' C_{w_x w_z},$$

$$E(e_1^2) = \theta C^2_{w_x}, E(e_2 e_4) = \theta\lambda_{021} C_{w_z}, E(e'_1 e'_4) = E(e_1 e'_4)$$

$$= E(e_4 e'_1) = \theta'\lambda_{030} C_{w_x},$$

$$E(e_2^2) = \theta C^2_{w_z}, E(e_1 e_4) = \theta\lambda_{030} C_{w_x}, E(e'_1 e'_5) = E(e_1 e'_5)$$

$$= E(e_5 e'_1) = \theta'\lambda_{012} C_{w_x},$$

$$E(e_3^2) = \theta\lambda^*_{400}, E(e_0 e_5) = \theta\lambda_{102} C_{w_y}, E(e'_2 e'_4) = E(e_2 e'_4)$$

$$= E(e_4 e'_2) = \theta'\lambda_{021} C_{w_z},$$

$$E(e_4^2) = \theta\lambda^*_{040}, E(e_0 e_3) = \theta\lambda_{300} C_{w_y}, E(e'_2 e'_5) = E(e_2 e'_5)$$

$$= E(e_5 e'_2) = \theta'\lambda_{003} C_{w_z},$$

$$E(e_5^2) = \theta\lambda^*_{004}, E(e_3 e_4) = \theta\lambda^*_{220}, E(e_3 e_5) = \theta\lambda^*_{202},$$

$$,E(e_4'e_5') = E(e_4e_5') = E(e_5e_4') = \theta'\lambda_{022}^*,$$

$$E(e_0e_1) = \theta C_{w_yw_x}, E(e_0e_2) = \theta C_{w_yw_z}, E(e_1e_5) = \theta\lambda_{012}C_{w_x},$$

$$,E(e_1'^2) = E(e_1e_1') = \theta' C_{w_x}^2,$$

$$E(e_1e_2) = \theta C_{w_xw_z}, E(e_0e_4) = \theta C_{w_y}\lambda_{120}, E(e_3e_5') = \theta'\lambda_{202}^*,$$

$$,E(e_2'^2) = E(e_2e_2') = \theta' C_{w_z}^2,$$

$$E(e_4e_5) = \theta\lambda_{022}^*, E(e_2e_5) = \theta\lambda_{003}C_{w_z}, E(e_3e_1') = \theta'\lambda_{210}C_{w_x},$$

$$,E(e_4'^2) = E(e_4e_4') = \theta'\lambda_{040}^*,$$

$$E(e_3e_2') = \theta'\lambda_{201}C_{w_z}, E(e_3e_4') = \theta'\lambda_{220}^*, E(e_1e_3) = \theta\lambda_{210}C_{w_x},$$

$$,E(e_5'^2) = E(e_5e_5') = \theta'\lambda_{004}^*,$$

where

$$\theta = \left(\frac{1}{n} - \frac{1}{N}\right), \mu_{rst} = \frac{1}{N-1}\sum_{i=1}^{N}\left(w_{yi} - \overline{Y}_w\right)^r\left(w_{xi} - \overline{X}_w\right)^s\left(w_{zi} - \overline{Z}_w\right)^t,$$

$$\theta' = \left(\frac{1}{n'} - \frac{1}{N}\right), \theta'' = \theta - \theta' = \left(\frac{1}{n} - \frac{1}{n'}\right), \lambda_{rst}^* = \lambda_{rst} - 1, \lambda_{rst}$$

$$= \frac{\mu_{rst}}{\mu_{200}^{\frac{r}{2}}\mu_{020}^{\frac{s}{2}}\mu_{002}^{\frac{t}{2}}}.$$

The general parameter of interest is defined by,

$$\tau(\alpha,\beta) = \overline{Y}_w^{\alpha} S_{w_y}^{\beta},\ \overline{Y}_w \neq 0,$$

where α and β are suitably chosen constants. For different values of α and β, $\tau(\alpha,\beta)$ reduces to the following population parameters:

1. $\alpha = 1, \beta = 0, \tau(\alpha,\beta) = \overline{Y}_w$ (population mean)
2. $\alpha = 0, \beta = 2, \tau(\alpha,\beta) = S_{w_y}^2$ (population variance)
3. $\alpha = -1, \beta = 1, \tau(\alpha,\beta) = C_{w_y}$ (population coefficient of variation)

Thompson (1990) proposed an unbiased estimator for population mean based on Hansen-Hurwitz estimator as,

$$t_{om} = \overline{w}_y = \frac{1}{n}\sum_{i=1}^{n}(w_y)_i, \quad ...(1)$$

where,

$$(w_y)_i = \frac{1}{m_i}\sum_{j\in\Psi_i} y_j,$$

Ψ_i is the network that includes unit i, $(w_y)_i$ is the average of y values and m_i is the size of that network. The variance of t_{om} is,

$$V(t_{om}) = \theta \overline{Y}_w^2 C_{wy}^2 \quad ...(2)$$

Thompson (1992) suggested an unbiased estimator for population variance as,

$$t_{ov} = s_{w_y}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(w_{y_i} - \overline{w}_y)^2. \quad ...(3)$$

The variance of t_{ov} is,

$$V(t_{ov}) = \theta S_{w_y}^4 \lambda_{400}^* \quad ...(4)$$

Similarly, an estimator for coefficient of variation can be defined as,

$$t_{oc} = \hat{C}_{w_y} = \frac{s_{w_y}}{\overline{w}_y} \quad ...(5)$$

The bias and mean square error (MSE) of t_{oc} to first order of approximation are,

$$Bias(t_{oc}) = \theta C_{w_y}(C_{w_y}^2 - \frac{1}{2}C_{w_y}\lambda_{300} - \frac{1}{8}\lambda_{400}^*) \quad ...(6)$$

and

$$MSE(t_{oc}) = C_{w_y}^2\theta(C_{w_y}^2 + \frac{1}{4}\lambda_{400}^* - C_{w_y}\lambda_{300}) \quad ...(7)$$

Thus, the usual estimator for the general parameter $\tau(\alpha,\beta)$ is,

$$t_o = \hat{\tau}_{(\alpha,\beta)} = \overline{w}_y^{\alpha} s_{w_y}^{\beta} \quad ...(8)$$

The bias and MSE of t_o, to first order of approximation are,

$$Bias(t_o) = \tau(\alpha,\beta)\theta[\frac{\alpha(\alpha-1)}{2}C_{w_y}^2 + \frac{\alpha\beta}{2}C_{w_y}\lambda_{300} + \frac{\beta(\beta-2)}{8}\lambda_{400}^*] \quad ...(9)$$

and

$$MSE(t_o) = \tau(\alpha,\beta)^2 \theta[\alpha^2 C_{w_y}^2 + \frac{\beta^2}{4}\lambda_{400}^* + \alpha\beta C_{w_y}\lambda_{300}] \quad ...(10)$$

Dryver and Chao (2007) proposed ratio estimator for population mean as,

$$t_{dc} = \bar{w}_y \frac{\bar{X}_w}{\bar{\bar{w}}_x} \quad ...(11)$$

The bias and MSE of t_{dc} to first order of approximation are,

$$Bias(t_{dc}) = \theta \bar{Y}_w (C_{wx}^2 - C_{w_y w_x}) \quad ...(12)$$

and

$$MSE(t_{dc}) = \theta \bar{Y}_w^2 (C_{wy}^2 + C_{wx}^2 - 2C_{w_y w_x}) \quad ...(13)$$

Following Dryver and Chao (2007), ratio estimator for general parameter $\tau_{(\alpha,\beta)}$, when $(\bar{X}_w)$ is unknown and $(\bar{Z}_w)$ is known, can be defined as,

$$t_{dc-1} = \hat{\tau}_{(\alpha,\beta)} \left(\frac{\bar{w}_x'}{\bar{\bar{w}}_x}\right)\left(\frac{\bar{w}_z'}{\bar{\bar{w}}_z}\right)\left(\frac{\bar{Z}_w}{\bar{\bar{w}}_z}\right), \quad ...(14)$$

The bias and MSE of t_{dc-1} to first order of approximation are,

$$Bias(t_{dc-1}) = \tau_{(\alpha,\beta)}[\theta\{3C_{wz}^2 - 2\alpha C_{w_y w_z} - \beta\lambda_{201}C_{w_z} + \frac{\alpha\beta}{2}C_{w_y}\lambda_{300} + \frac{\alpha(\alpha-1)}{2}C_{wy}^2 + \frac{\beta(\beta-2)}{8}\lambda_{400}^*\} + \theta'\{\alpha C_{w_y w_z} + \frac{\beta}{2}\lambda_{201}C_{w_z} - 2C_{wz}^2\} + \theta''\{C_{wx}^2 + 2C_{w_z w_x} - \alpha C_{w_y w_x} - \frac{\beta}{2}\lambda_{210}C_{w_x}\}] \quad ...(15)$$

and

$$MSE(t_{dc-1}) = \tau_{(\alpha,\beta)}^2[\theta\{\alpha^2 C_{wy}^2 + \frac{\beta^2}{4}\lambda_{400}^* - 2\beta\lambda_{201}C_{w_z} + \frac{\alpha\beta}{2}C_{w_y}\lambda_{300} + 4C_{wz}^2 - 4\alpha C_{w_y w_z}\} + \theta'\left\{2\alpha C_{w_y w_z} + \beta\lambda_{201}C_{w_z} - 3C_{wz}^2\right\} + \theta''\left\{C_{wx}^2 + 4C_{w_z w_x} - 2\alpha C_{w_y w_x} - \beta\lambda_{210}C_{w_x}\right\}] \quad ...(16)$$

Similarly, ratio estimator for general parameter $\tau_{(\alpha,\beta)}$, when both $(\bar{X}_w, \bar{Z}_w)$ are unknown, can be defined as,

$$t_{dc-2} = \hat{\tau}_{(\alpha,\beta)} \left(\frac{\bar{w}_x'}{\bar{\bar{w}}_x}\right)\left(\frac{\bar{w}_z'}{\bar{\bar{w}}_z}\right) \quad ...(17)$$

The bias and MSE of t_{dc-2}, to first order of approximation are,

$$Bias(t_{dc-2}) = \tau_{(\alpha,\beta)}[\theta\{\frac{\alpha\beta}{2}C_{w_y}\lambda_{300} + \frac{\alpha(\alpha-1)}{2}C_{wy}^2 + \frac{\beta(\beta-2)}{8}\lambda_{400}^*\} + \theta''\{C_{wx}^2 + C_{wz}^2 + C_{w_x w_z} - \frac{\beta}{2}\lambda_{210}C_{w_x} - \frac{\beta}{2}\lambda_{201}C_{w_z} - \alpha C_{w_y w_x} - \alpha C_{w_y w_z}\}] \quad ...(18)$$

and

$$MSE(t_{dc-2}) = \tau_{(\alpha,\beta)}^2[\theta\{\alpha^2 C_{wy}^2 + \frac{\beta^2}{4}\lambda_{400}^* + \alpha\beta C_{w_y}\lambda_{300}\} + \theta''\{C_{wx}^2 + C_{wz}^2 + 2C_{w_x w_z} - 2\alpha C_{w_y w_x} - 2\alpha C_{w_y w_z} - \beta\lambda_{210}C_{w_x} - \beta\lambda_{201}C_{wz}\}] \quad ...(19)$$

Also, ratio estimator for general parameter $\tau_{(\alpha,\beta)}$, when both $(\bar{X}_w, \bar{Z}_w)$ are known, can be defined as,

$$t_{dc-3} = \hat{\tau}_{(\alpha,\beta)} \left(\frac{\bar{X}_w}{\bar{\bar{w}}_x}\right)\left(\frac{\bar{Z}_w}{\bar{\bar{w}}_z}\right) \quad ...(20)$$

The bias and MSE of t_{dc-3}, to first order of approximation are,

$$Bias(t_{dc-3}) = \tau_{(\alpha,\beta)}\theta\{C_{wx}^2 + C_{wz}^2 - \alpha C_{w_y w_x} + \frac{\alpha(\alpha-1)}{2}C_{wy}^2 + \frac{\beta(\beta-2)}{8}\lambda_{400}^* + \frac{\alpha\beta}{2}C_{w_y}\lambda_{300}$$

$$+C_{w_xw_z} - \alpha C_{w_yw_z} - \frac{\beta}{2}\lambda_{210}C_{w_x} - \frac{\beta}{2}\lambda_{201}C_{w_z}\} \quad ...(21)$$

and

$$MSE(t_{dc-3}) = \tau^2_{(\alpha,\beta)}\theta\{C^2_{wx}+C^2_{wz}+\alpha^2C^2_{wy}+\frac{\beta^2}{4}\lambda^*_{400}$$

$$+\alpha\beta C_{w_y}\lambda_{300}+2C_{w_xw_z}-2\alpha C_{w_yw_x}$$

$$-2\alpha C_{w_yw_z} - \beta\lambda_{210}C_{w_x} - \beta\lambda_{201}C_{wz}\} \quad ...(22)$$

METHODOLOGY

Estimator for general parameter when ($\overline{X}_w, S^2_{w_x}$) are unknown and ($\overline{Z}_w, S^2_{w_z}$) are known

Motivated by Shabbir and Gupta (2017), the following exponential ratio type estimator for the population parameter is proposed:

$$t_1 = \hat{\tau}_{(\alpha,\beta)}\exp\left\{\frac{\overline{w}'_x - \overline{w}_x}{\overline{w}'_x + (a-1)\overline{w}_x}\right\}\exp\left\{\frac{\overline{w}'_z - \overline{w}_z}{\overline{w}'_z + (c-1)\overline{w}_z}\right\}$$

$$\exp\left\{\frac{\overline{Z}_w - \overline{w}_z}{\overline{Z}_w + (g-1)\overline{w}_z}\right\}\exp\left\{\frac{s^{2'}_{w_x} - s^2_{w_x}}{s^{2'}_{w_x} + (b-1)s^2_{w_x}}\right\}$$

$$\exp\left\{\frac{s^{2'}_{w_z} - s^2_{w_z}}{s^{2'}_{w_z} + (d-1)s^2_{w_z}}\right\}\exp\left\{\frac{S^2_{w_z} - s^2_{w_z}}{S^2_{w_z} + (f-1)s^2_{wz}}\right\}, \quad ...(23)$$

where a, b, c, d, g, f are suitably chosen constants. Rewriting equation (23) in terms of error terms and considering first order approximation, we have,

$$t_1 = \tau_{(\alpha,\beta)}\{1+\alpha e_0 + \frac{\beta}{2}e_3 + \frac{\alpha\beta}{2}e_0e_3 + \frac{\alpha(\alpha-1)}{2}e_0^2 +$$

$$\frac{\beta(\beta-2)}{8}e_3^2\}\{1+\frac{e'_4}{b} - \frac{e_4}{b} + \frac{e'_1}{a} - \frac{e_1}{a} - \frac{e'^2_4}{2b^2} +$$

$$\frac{2b-1}{2b^2}e_4^2 - \frac{e'^2_1}{2a^2} + \frac{2a-1}{2a^2}e_1^2 + \frac{1-b}{b^2}e_4e'_4 +$$

$$\frac{1-a}{a^2}e_1e'_1 + \frac{e'_1e'_4}{ab} - \frac{e'_1e_4}{ab} - \frac{e_1e'_4}{ab} + \frac{e_1e_4}{ab}\}$$

$$\}\{1+\frac{e'_5}{d} - \frac{e_5}{d} + \frac{e'_2}{c} - \frac{e_2}{c} - \frac{e'^2_5}{2d^2} + \frac{2d-1}{2d^2}e_5^2 +$$

$$\frac{2c-1}{2c^2}e_2^2 - \frac{e'^2_2}{2c^2} + \frac{1-d}{d^2}e_5e'_5 + \frac{1-c}{c^2}e_2e'_2 +$$

$$\frac{e'_2e'_5}{cd} - \frac{e'_2e_5}{cd} - \frac{e_2e'_5}{cd} + \frac{e_2e_5}{cd}\}\{1 - \frac{e_5}{f} - \frac{e_2}{g}$$

$$+\frac{2f-1}{2f^2}e_5^2 + \frac{2g-1}{2g^2}e_2^2 + \frac{e_2e_5}{gf}\} \quad ...(24)$$

Multiplying out the terms on right hand side, we have,

$$t_1 = \tau_{(\alpha,\beta)}[1+\frac{e'_5}{d} - \frac{e_5}{d} + \frac{e'_2}{c} - \frac{e_2}{c} + \frac{e'_4}{b} - \frac{e_4}{b} + \frac{e'_1}{a} - \frac{e_1}{a}$$

$$+\alpha e_0 + \frac{\beta}{2}e_3 - \frac{e_5}{f} - \frac{e_2}{g} + (\frac{2d-1}{2d^2} + \frac{1}{df})(e_5^2 - e'^2_5)$$

$$+(\frac{2c-1}{2c^2} + \frac{1}{cg})(e_2^2 - e'^2_2) + (\frac{2f-1}{2f^2})e_5^2 +$$

$$\frac{2b-1}{2b^2}(e_4^2 - e'^2_4) + (\frac{1}{cd} + \frac{1}{cf} + \frac{1}{dg})(e_2e_5 - e_2e'_5) +$$

$$(\frac{2g-1}{2g^2})e_2^2 + \frac{1}{gf}e_2e_5 + (\frac{1}{bd} + \frac{1}{bf})(e_4e_5 - e_4e'_5)$$

$$+\frac{2a-1}{2a^2}(e_1^2 - e'^2_1) + (\frac{1}{bc} + \frac{1}{bg})(e_2e_4 - e_2e'_4) +$$

$$\frac{\alpha(\alpha-1)}{2}e_0^2 + \frac{1}{ab}(e_1e_4 - e_1e'_4)$$

$$+(\frac{1}{ad} + \frac{1}{af})(e_1e_5 - e_1e'_5) - \frac{\alpha}{g}e_0e_2 - \frac{\alpha}{f}e_0e_5 +$$

$$+(\frac{1}{ac} + \frac{1}{ag})(e_1e_2 - e_1e'_2) - \frac{\beta}{2f}e_3e_5$$

$$+\frac{\beta(\beta-2)}{8}e_3^2 - \frac{\alpha}{d}(e_0e_5 - e_0e'_5) - \frac{\alpha}{c}(e_0e_2 - e_0e'_2)$$

$$-\frac{\alpha}{b}(e_0e_4 - e_0e'_4) - \frac{\beta}{2g}e_3e_2 - \frac{\alpha}{a}(e_0e_1 - e_0e'_1)$$

$$-\frac{\beta}{2d}(e_3e_5 - e_3e'_5) - \frac{\beta}{2c}(e_3e_2 - e_3e'_2) -$$

$$\frac{\beta}{2b}(e_3e_4 - e_3e'_4) - \frac{\beta}{2a}(e_3e_1 - e_3e'_1) + \frac{\alpha\beta}{2}e_0e_3] \quad ...(25)$$

From equation (25), we get the bias of proposed estimator t_1 as,

$$
\begin{aligned}
Bias(t_1) = \tau_{(\alpha,\beta)}[\theta\{ & \frac{w_6(2-w_6)}{2}\lambda^*_{004} + \frac{w_5(2-w_5)}{2}C^2_{w_z} \\
& + w_5 w_6 \lambda_{003} C_{w_z} + \frac{\alpha\beta}{2} C_{w_y}\lambda_{300} + \frac{\alpha(\alpha-1)}{2}C^2_{w_y} + \\
& \frac{\beta(\beta-2)}{8}\lambda^*_{400} - \alpha w_6 \lambda_{102} C_{w_y} - \alpha w_5 C_{w_y w_z} - \frac{\beta}{2} w_6 \lambda^*_{202} \\
& - \frac{\beta}{2} w_5 \lambda_{201} C_{w_z}\} + \theta''\{\frac{w_1(2-w_1)}{2}C^2_{w_x} + \\
& (w_3 w_4 + w_3 w_6 + w_4 w_5)\lambda_{003} C_{w_z} + (\frac{w_4(2-w_4)}{2} + \\
& w_4 w_6)\lambda^*_{004} + (w_1 w_4 + w_1 w_6)\lambda_{012} C_{w_x} - \alpha w_2 \lambda_{120} C_{w_y} \\
& + \frac{w_2(2-w_2)}{2}\lambda^*_{040} + (w_2 w_4 + w_2 w_6)\lambda^*_{022} + \\
& w_1 w_2 \lambda_{030} C_{w_x} - \frac{\beta}{2} w_1 \lambda_{210} C_{w_x} + (\frac{w_3(2-w_3)}{2} + w_3 w_5)C^2_{w_z} \\
& + (w_2 w_3 + w_2 w_5)\lambda_{021} C_{w_z} - \frac{\beta}{2} w_3 \lambda_{201} C_{w_z} \\
& + (w_1 w_3 + w_1 w_5) C_{w_x w_z} - \alpha w_4 \lambda_{102} C_{w_y} - \alpha w_3 C_{w_y w_z} \\
& - \alpha w_1 C_{w_y w_x} - \frac{\beta}{2} w_4 \lambda^*_{202} - \frac{\beta}{2} w_2 \lambda^*_{220}\}]
\end{aligned}
\quad ...(26)
$$

where $w_1 = \frac{1}{a}, w_2 = \frac{1}{b}, w_3 = \frac{1}{c}, w_4 = \frac{1}{d}, w_5 = \frac{1}{g}$, $w_6 = \frac{1}{f}$. The mean square error of the proposed estimator t_1, to the first degree of approximation is obtained as,

$$
\begin{aligned}
MSE(t_1) = \tau^2_{(\alpha,\beta)}[\theta\{ & \alpha^2 C^2_{w_y} + w_5^2 C^2_{w_z} + \frac{\beta^2}{4}\lambda^*_{400} + 2 w_5 w_6 \lambda_{003} C_{w_z} - 2\alpha w_6 \lambda_{102} C_{w_y} + w_6^2 \lambda^*_{004} \\
& - 2\alpha w_5 C_{w_y w_z} - \beta w_6 \lambda^*_{202} - \beta w_5 \lambda_{201} C_{w_z} + \alpha\beta C_{w_y}\lambda_{300}\} + \theta''\{w_1^2 C^2_{w_x} + w_2^2 \lambda^*_{040} \\
& + C^2_{w_z}(w_3^2 + 2 w_3 w_5) + \lambda^*_{004}(w_4^2 + 2 w_4 w_6) + 2\lambda_{003} C_{w_z}(w_3 w_4 + w_3 w_6 + w_4 w_5) \\
& + 2\lambda^*_{022}(w_2 w_4 + w_2 w_6) + 2\lambda_{021} C_{w_z}(w_2 w_3 + w_2 w_5) + 2 C_{w_x w_z}(w_1 w_3 + w_1 w_5) \\
& + 2\lambda_{012} C_{w_x}(w_1 w_4 + w_1 w_6) + 2 w_1 w_2 \lambda_{030} C_{w_x} - 2\alpha w_4 \lambda_{102} C_{w_y} - 2\alpha w_1 C_{w_y w_x} \\
& - 2\alpha w_3 C_{w_y w_z} - 2\alpha w_2 \lambda_{120} C_{w_y} - \beta w_4 \lambda^*_{202} - \beta w_3 \lambda_{201} C_{w_z} - \beta w_2 \lambda^*_{220} \\
& - \beta w_1 \lambda_{210} C_{w_x}\}]
\end{aligned}
\quad ...(27)
$$

We get minimum mean square error of t_1 by substituting the optimum values of $w_i \forall i = 1,2,...,6$. Optimum values of $w_i \forall i = 1,2,...,6$ are obtained by solving

$$AX = B \quad ...(28)$$

where

$$
A = \begin{bmatrix}
2C_{w_x} & 2\lambda_{030} & 2\rho_{w_x w_z} C_{w_z} & 2\lambda_{012} & 2\rho_{w_x w_z} C_{w_z} & 2\lambda_{012} \\
2\lambda_{030} C_{w_x} & 2\lambda^*_{040} & 2\lambda_{021} C_{w_z} & 2\lambda^*_{022} & 2\lambda_{021} C_{w_z} & 2\lambda^*_{022} \\
2\rho_{w_x w_z} C_{w_x} & 2\lambda_{021} & 2C_{w_z} & 2\lambda_{003} & 2C_{w_z} & 2\lambda_{003} \\
2\lambda_{012} C_{w_x} & 2\lambda^*_{022} & 2\lambda_{003} C_{w_z} & 2\lambda^*_{004} & 2\lambda_{003} C_{w_z} & 2\lambda^*_{004} \\
2\rho_{w_x w_z} C_{w_x}\theta'' & 2\lambda_{021}\theta'' & 2C_{w_z}\theta'' & 2\lambda_{003}\theta'' & 2C_{w_z}\theta & 2\lambda_{003}\theta \\
2\lambda_{012} C_{w_x}\theta'' & 2\lambda^*_{022}\theta'' & 2\lambda_{003} C_{w_z}\theta'' & 2\lambda^*_{004}\theta'' & 2\lambda_{003} C_{w_z}\theta & 2\lambda^*_{004}\theta
\end{bmatrix},
$$

$$
X = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix},
B = \begin{bmatrix}
2\alpha\rho_{w_x w_y} C_{wy} + \beta\lambda_{210} \\
2\alpha\lambda_{120} C_{w_y} + \beta\lambda^*_{220} \\
2\alpha\rho_{w_z w_y} C_{wy} + \beta\lambda_{201} \\
2\alpha\lambda_{102} C_{w_y} + \beta\lambda^*_{202} \\
2\alpha\rho_{w_z w_y} C_{wy}\theta + \beta\lambda_{201}\theta \\
2\alpha\lambda_{102} C_{w_y}\theta + \beta\lambda^*_{202}\theta
\end{bmatrix}
$$

The exponential ratio type estimator for population mean t_{1m} can be obtained by substituting $\alpha = 1, \beta = 0$ in equation (23). The bias and MSE of t_{1m} can be obtained by putting $\alpha = 1, \beta = 0$ in equations (26) and (27). Similarly, exponential ratio type estimator for population variance (t_{1v}) and coefficient of variation (t_{1c}) can be obtained by putting $\alpha = 0, \beta = 2$ and $\alpha = -1, \beta = 1$ respectively in equation (23).

Estimator for general parameter when all ($\overline{X}_w, S^2_{w_x}$) and ($\overline{Z}_w, S^2_{w_z}$) are unknown

When $\overline{X}_w, S^2_{w_x}$ and $\overline{Z}_w, S^2_{w_z}$ all are unknown, the following estimator for general parameter is proposed:

$$t_2 = \hat{\tau}_{(\alpha,\beta)} \exp\left\{\frac{\overline{w}_x' - \overline{w}_x}{\overline{w}_x' + (a-1)\overline{w}_x}\right\} \exp\left\{\frac{\overline{w}_z' - \overline{w}_z}{\overline{w}_z' + (c-1)\overline{w}_z}\right\} \exp\left\{\frac{s^{2'}_{w_x} - s^2_{w_x}}{s^{2'}_{w_x} + (b-1)s^2_{w_x}}\right\} \exp\left\{\frac{s^{2'}_{w_z} - s^2_{w_z}}{s^{2'}_{w_z} + (d-1)s^2_{w_z}}\right\}, \quad ...(29)$$

where a, b, c, d are suitably chosen constants. Rewriting equation (29) in terms of error terms and considering first order approximation, we have,

$$t_2 = \tau_{(\alpha,\beta)}\{1 + \alpha e_0 + \frac{\beta}{2}e_3 + \frac{\alpha\beta}{2}e_0 e_3 + \frac{\alpha(\alpha-1)}{2}e_0^2 + \frac{\beta(\beta-2)}{8}e_3^2\}\{1 + \frac{e_4'}{b} - \frac{e_4}{b} + \frac{e_1'}{a} - \frac{e_1}{a} - \frac{e_4'^2}{2b^2} + \frac{2b-1}{2b^2}e_4^2 -$$

$$\frac{e_1'^2}{2a^2} + \frac{2a-1}{2a^2}e_1^2 + \frac{1-b}{b^2}e_4 e_4' + \frac{e_1' e_4'}{ab} + \frac{e_1 e_4}{ab} + \frac{1-a}{a^2}e_1 e_1' - \frac{e_1' e_4}{ab} - \frac{e_1 e_4'}{ab}\}\{1 + \frac{e_5'}{d} - \frac{e_5}{d} + \frac{e_2'}{c} - \frac{e_2}{c} - \frac{e_5'^2}{2d^2} +$$

$$\frac{2d-1}{2d^2}e_5^2 + \frac{1-d}{d^2}e_5 e_5' + \frac{1-c}{c^2}e_2 e_2' - \frac{e_2'^2}{2c^2} + \frac{2c-1}{2c^2}e_2^2 + \frac{e_2' e_5'}{cd} - \frac{e_2' e_5}{cd} - \frac{e_2 e_5'}{cd} + \frac{e_2 e_5}{cd}\} \quad ...(30)$$

Multiplying out the terms on right hand side, we have,

$$t_2 = \tau_{(\alpha,\beta)}[1 + \frac{e_5'}{d} - \frac{e_5}{d} + \frac{e_2'}{c} - \frac{e_2}{c} + \frac{e_4'}{b} - \frac{e_4}{b} + \frac{e_1'}{a} - \frac{e_1}{a} + \alpha e_0 + \frac{\alpha(\alpha-1)}{2}e_0^2 + \frac{\beta}{2}e_3 + \frac{\beta(\beta-2)}{8}e_3^2 +$$

$$\frac{2a-1}{2a^2}(e_1^2 - e_1'^2) + \frac{2b-1}{2b^2}(e_4^2 - e_4'^2) + \frac{\alpha\beta}{2}e_0 e_3 + \frac{2c-1}{2c^2}(e_2^2 - e_2'^2) + \frac{2d-1}{2d^2}(e_5^2 - e_5'^2) + \frac{1}{cd}(e_2 e_5 - e_2 e_5') +$$

$$\frac{1}{bd}(e_4 e_5 - e_4 e_5') + \frac{1}{bc}(e_2 e_4 - e_2 e_4') + \frac{1}{ad}(e_1 e_5 - e_1 e_5') + \frac{1}{ac}(e_1 e_2 - e_1 e_2') + \frac{1}{ab}(e_1 e_4 - e_1 e_4')$$

$$-\frac{\alpha}{d}(e_0 e_5 - e_0 e_5') - \frac{\alpha}{c}(e_0 e_2 - e_0 e_2') - \frac{\beta}{2a}(e_3 e_1 - e_3 e_1') - \frac{\beta}{2b}(e_3 e_4 - e_3 e_4')$$

$$-\frac{\alpha}{a}(e_0 e_1 - e_0 e_1') - \frac{\alpha}{b}(e_0 e_4 - e_0 e_4') - \frac{\beta}{2c}(e_3 e_2 - e_3 e_2') - \frac{\beta}{2d}(e_3 e_5 - e_3 e_5')] \quad ...(31)$$

From equation (31), we get the bias of proposed estimator t_2 as,

$$Bias(t_2) = \tau_{(\alpha,\beta)}[\theta\{\frac{\alpha(\alpha-1)}{2}C^2_{w_y} + \frac{\beta(\beta-2)}{8}\lambda^*_{400} + \frac{\alpha\beta}{2}C_{w_y}\lambda_{300}\} + \theta''\{w_1w_3C_{w_xw_z} + \frac{w_4(2-w_4)}{2}\lambda^*_{004} +$$

$$\frac{w_1(2-w_1)}{2}C^2_{w_x} + \frac{w_3(2-w_3)}{2}C^2_{w_z} + w_3w_4\lambda_{003}C_{w_z} + w_1w_4\lambda_{012}C_{w_x} + w_1w_2\lambda_{030}C_{w_x} -$$

$$-\alpha w_1C_{w_yw_x} - \alpha w_2\lambda_{120}C_{w_y} - \alpha w_3C_{w_yw_z} + \frac{w_2(2-w_2)}{2}\lambda^*_{040} + w_2w_3\lambda_{021}C_{w_z} + w_2w_4\lambda^*_{022} -$$

$$\frac{\beta}{2}w_4\lambda^*_{202} - \frac{\beta}{2}w_2\lambda^*_{220} - \frac{\beta}{2}w_3\lambda_{201}C_{w_z} - \alpha w_4\lambda_{102}C_{w_y} - \frac{\beta}{2}w_1\lambda_{210}C_{w_x}\}] \quad ...(32)$$

The MSE of proposed estimator t_2, to the first degree of approximation is obtained as,

$$MSE(t_2) = \tau^2_{(\alpha,\beta)}[\theta\{\alpha^2C^2_{w_y} + \frac{\beta^2}{4}\lambda^*_{400} + \alpha\beta C_{w_y}\lambda_{300}\} + \theta''\{w_4^2\lambda^*_{004} + w_3^2C^2_{w_z} + 2w_2w_4\lambda^*_{022} + w_1^2C^2_{w_x} +$$

$$2w_3w_4\lambda_{003}C_{w_z} - 2\alpha w_1C_{w_yw_x} + 2w_1w_4\lambda_{012}C_{w_x} + 2w_1w_2\lambda_{030}C_{w_x} + w_2^2\lambda^*_{040} + 2w_2w_3\lambda_{021}C_{w_z} +$$

$$2w_1w_3C_{w_xw_z} - 2\alpha w_4\lambda_{102}C_{w_y} - 2\alpha w_2\lambda_{120}C_{w_y} - 2\alpha w_3C_{w_yw_z} - \beta w_1\lambda_{210}C_{w_x} - \beta w_2\lambda^*_{220} -$$

$$\beta w_3\lambda_{201}C_{w_z} - \beta w_4\lambda^*_{202}\}] \quad ...(33)$$

The exponential ratio type estimator for population mean t_{2m} can be obtained by substituting $\alpha = 1, \beta = 0$ in equation (29). The bias and MSE of t_{2m} can be obtained by putting $\alpha = 1, \beta = 0$ in equations (32) and (33). Similarly, exponential ratio type estimator for population variance (t_{2v}) and coefficient of variation (t_{2c}) can be obtained by putting $\alpha = 0, \beta = 2$ and $\alpha = -1, \beta = 1$ respectively in equation (29).

Estimator for general parameter when all ($\bar{X}_w, S^2_{w_x}$) and ($\bar{Z}_w, S^2_{w_z}$) are known

When $\bar{X}_w, S^2_{w_x}$ and $\bar{Z}_w, S^2_{w_z}$ all are known, we propose the following estimator for general parameter:

$$t_3 = \hat{\tau}_{(\alpha,\beta)} \exp\left\{\frac{\bar{X}_w - \bar{w}_x}{\bar{X}_w + (a-1)\bar{w}_x}\right\} \exp\left\{\frac{\bar{Z}_w - \bar{w}_z}{\bar{Z}_w + (c-1)\bar{w}_z}\right\}$$

$$\exp\left\{\frac{S^2_{w_x} - s^2_{w_x}}{S^2_{w_x} + (b-1)s^2_{w_x}}\right\} \exp\left\{\frac{S^2_{w_z} - s^2_{w_z}}{S^2_{w_z} + (d-1)s^2_{w_z}}\right\} \quad ...(34)$$

where a, b, c, d are suitably chosen constants. Rewriting equation (34) in terms of error terms and considering first order approximation, we have,

$$t_3 = \tau_{(\alpha,\beta)}\{1 + \alpha e_0 + \frac{\beta}{2}e_3 + \frac{\alpha\beta}{2}e_0e_3 + \frac{\alpha(\alpha-1)}{2}e_0^2 +$$

$$\frac{\beta(\beta-2)}{8}e_3^2\}\{1 - \frac{e_4}{b} - \frac{e_1}{a} + \frac{2b-1}{2b^2}e_4^2 + \frac{2a-1}{2a^2}e_1^2 + \frac{e_1e_4}{ab}\}$$

$$\}\{1 - \frac{e_5}{d} - \frac{e_2}{c} + \frac{2d-1}{2d^2}e_5^2 + \frac{2c-1}{2c^2}e_2^2 + \frac{e_2e_5}{cd}\} \quad ...(35)$$

Multiplying out the terms on right hand side, we have,

$$t_3 = \tau_{(\alpha,\beta)}[1 - \frac{e_5}{d} - \frac{e_2}{c} - \frac{e_4}{b} - \frac{e_1}{a} + \alpha e_0 + \frac{\beta}{2}e_3 - \frac{\alpha}{d}e_0e_5 - \frac{\alpha}{c}e_0e_2$$

$$+ \frac{2d-1}{2d^2}e_5^2 - \frac{\alpha}{b}e_0e_4 - \frac{\alpha}{a}e_0e_1 + \frac{2c-1}{2c^2}e_2^2 + \frac{2b-1}{2b^2}e_4^2 + \frac{2a-1}{2a^2}e_1^2$$

$$+ \frac{\alpha\beta}{2}e_0e_3 + \frac{1}{cd}e_2e_5 + \frac{1}{bd}e_4e_5 + \frac{\alpha(\alpha-1)}{2}e_0^2 + \frac{1}{bc}e_2e_4 + \frac{1}{ad}e_1e_5$$

$$+ \frac{1}{ac}e_1e_2 + \frac{1}{ab}e_1e_4 - \frac{\beta}{2a}e_3e_1 + \frac{\beta(\beta-2)}{8}e_3^2 - \frac{\beta}{2c}e_3e_2$$

$$- \frac{\beta}{2b}e_3e_4 - \frac{\beta}{2d}e_3e_5] \quad ...(36)$$

From equation (36), we get the bias of proposed estimator t_3 as,

$$Bias(t_3)=\tau_{(\alpha,\beta)}\theta[\frac{w_1(2-w_1)}{2}C^2_{w_x}+\frac{w_2(2-w_2)}{2}\lambda^*_{040}+\frac{w_3(2-w_3)}{2}C^2_{w_z}+w_1w_3C_{w_xw_z}$$
$$+\frac{w_4(2-w_4)}{2}\lambda^*_{004}+\frac{\alpha(\alpha-1)}{2}C^2_{w_y}+w_1w_4\lambda_{012}C_{w_x}+w_2w_4\lambda^*_{022}-\alpha w_1C_{w_yw_x}$$
$$+\frac{\beta(\beta-2)}{8}\lambda^*_{400}+\frac{\alpha\beta}{2}C_{w_y}\lambda_{300}+w_2w_3\lambda_{021}C_{w_z}+w_3w_4\lambda_{003}C_{w_z}-\alpha w_3C_{w_yw_z}$$
$$+w_1w_2\lambda_{030}C_{w_x}-\alpha w_2\lambda_{120}C_{w_y}-\alpha w_4\lambda_{102}C_{w_y}-\frac{\beta}{2}w_2\lambda^*_{220}-\frac{\beta}{2}w_1\lambda_{210}C_{w_x}$$
$$-\frac{\beta}{2}w_4\lambda^*_{202}-\frac{\beta}{2}w_3\lambda_{201}C_{w_z}] \quad ...(37)$$

The mean square error of the proposed estimator t_3, to the first degree of approximation is obtained as

$$MSE(t_3)=\tau^2_{(\alpha,\beta)}\theta[\alpha^2C^2_{w_y}+\frac{\beta^2}{4}\lambda^*_{400}+\alpha\beta C_{w_y}\lambda_{300}+w_2^2\lambda^*_{040}+w_1^2C^2_{w_x}+w_3^2C^2_{w_z}+w_4^2\lambda^*_{004}$$
$$+2w_3w_4\lambda_{003}C_{w_z}+2w_2w_4\lambda^*_{022}+2w_1w_4\lambda_{012}C_{w_x}-2\alpha w_1C_{w_yw_x}-2\alpha w_3C_{w_yw_z}$$
$$+2w_1w_2\lambda_{030}C_{w_x}+2w_1w_3C_{w_xw_z}-2\alpha w_2\lambda_{120}C_{w_y}-\beta w_1\lambda_{210}C_{w_x}-\beta w_2\lambda^*_{220}$$
$$+2w_2w_3\lambda_{021}C_{w_z}-2\alpha w_4\lambda_{102}C_{w_y}-\beta w_3\lambda_{201}C_{w_z}-\beta w_4\lambda^*_{202}] \quad ...(38)$$

Optimum values of $w_i \forall i=1,2,...,4$ for t_2 and t_3 can be obtained by solving the following system of equations:

$$\begin{bmatrix} 2C_{w_x} & 2\lambda_{030} & 2\rho_{w_xw_z}C_{w_z} & 2\lambda_{012} \\ 2\lambda_{030}C_{w_x} & 2\lambda^*_{040} & 2\lambda_{021}C_{w_z} & 2\lambda^*_{022} \\ 2\rho_{w_xw_z}C_{w_x} & 2\lambda_{021} & 2C_{w_z} & 2\lambda_{003} \\ 2\lambda_{012}C_{w_x} & 2\lambda^*_{022} & 2\lambda_{003}C_{w_z} & 2\lambda^*_{004} \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}=\begin{bmatrix} 2\alpha\rho_{w_xw_y}C_{wy}+\beta\lambda_{210} \\ 2\alpha\lambda_{120}C_{w_y}+\beta\lambda^*_{220} \\ 2\alpha\rho_{w_zw_y}C_{wy}+\beta\lambda_{201} \\ 2\alpha\lambda_{102}C_{w_y}+\beta\lambda^*_{202} \end{bmatrix}$$

The exponential ratio type estimator for population mean t_{3m} can be obtained by substituting $\alpha=1,\beta=0$ in equation (34). The bias and MSE of t_{3m} can be obtained by putting $\alpha=1,\beta=0$ in equations (37) and (38). Similarly, the exponential ratio type estimator for population variance (t_{3v}) and coefficient of variation (t_{3c}) can be obtained by putting $\alpha=0,\beta=2$ and $\alpha=-1,\beta=1$ respectively in equation (34).

Efficiency comparison

Comparison of the proposed estimators with other estimators are considered here.

From equations (10) and (27), $MSE(t_1)<MSE(t_0)$ if

$$\theta\{w_6^2\lambda^*_{004}+w_5^2C^2_{w_z}+2w_5w_6\lambda_{003}C_{w_z}-2\alpha w_6\lambda_{102}C_{w_y}-2\alpha w_5C_{w_yw_z}-\beta w_5\lambda_{201}C_{w_z}-\beta w_6\lambda^*_{202}$$
$$\}+\theta''\{\lambda^*_{004}(w_4^2+2w_4w_6)+2\lambda_{003}C_{w_z}(w_3w_4+w_3w_6+w_4w_5)+C^2_{w_z}(w_3^2+2w_3w_5)+w_1^2C^2_{w_x}$$
$$+2w_1w_2\lambda_{030}C_{w_x}+w_2^2\lambda^*_{040}+2\lambda_{012}C_{w_x}(w_1w_4+w_1w_6)+2(w_2w_4+w_2w_6)\lambda^*_{022}-\beta w_4\lambda^*_{202}$$
$$+2(w_2w_3+w_2w_5)\lambda_{021}C_{w_z}+2(w_1w_3+w_1w_5)C_{w_xw_z}-2\alpha w_2\lambda_{120}C_{w_y}-2\alpha w_4\lambda_{102}C_{w_y}$$
$$-2\alpha w_3C_{w_yw_z}-2\alpha w_1C_{w_yw_x}-\beta w_3\lambda_{201}C_{w_z}-\beta w_2\lambda^*_{220}-\beta w_1\lambda_{210}C_{w_x}\}<0 \quad ...(39)$$

From equations (16) and (27), $MSE(t_1) < MSE(t_{dc-1})$ if

$$\begin{aligned}
&\theta\{w_6^2\lambda_{004}^* + C_{w_z}^2(w_5^2-4) + 2w_5w_6\lambda_{003}C_{w_z} - \beta w_6\lambda_{202}^* - 2\alpha w_6\lambda_{102}C_{w_y} - \beta\lambda_{201}C_{w_z}(w_5-2) \\
&-C_{w_yw_z}(2\alpha w_5 - 4\alpha)\} - \theta'\left\{2\alpha C_{w_yw_z} + \beta\lambda_{201}C_{w_z} - 3C_{wz}^2\right\} + \theta''\{2(w_2w_3 + w_2w_5)\lambda_{021}C_{w_z} \\
&+2\lambda_{003}C_{w_z}(w_3w_4 + w_3w_6 + w_4w_5) + (w_3^2 + 2w_3w_5)C_{w_z}^2 + C_{w_x}^2(w_1^2-1) + 2w_1w_2\lambda_{030}C_{w_x} \\
&+(w_4^2 + 2w_4w_6)\lambda_{004}^* + 2(w_2w_4 + w_2w_6)\lambda_{022}^* + 2(w_1w_4 + w_1w_6)\lambda_{012}C_{w_x} - 2\alpha w_4\lambda_{102}C_{w_y} \\
&+w_2^2\lambda_{040}^* + 2(w_1w_3 + w_1w_5 - 4)C_{w_xw_z} - 2\alpha w_3C_{w_yw_z} - 2\alpha C_{w_yw_x}(w_1-1) - 2\alpha w_2\lambda_{120}C_{w_y} \\
&-\beta w_2\lambda_{220}^* - \beta w_4\lambda_{202}^* - \beta w_3\lambda_{201}C_{w_z} - \beta\lambda_{210}C_{w_x}(w_1-1)\} < 0
\end{aligned} \quad ...(40)$$

From equations (10) and (33), $MSE(t_2) < MSE(t_0)$ if

$$\begin{aligned}
&w_4^2\lambda_{004}^* + w_3^2C_{w_z}^2 + w_2^2\lambda_{040}^* + w_1^2C_{w_x}^2 + 2w_1w_4\lambda_{012}C_{w_x} + 2w_3w_4\lambda_{003}C_{w_z} + 2w_2w_3\lambda_{021}C_{w_z} \\
&+2w_2w_4\lambda_{022}^* + 2w_1w_3C_{w_xw_z} - 2\alpha w_1C_{w_yw_x} + 2w_1w_2\lambda_{030}C_{w_x} - 2\alpha w_4\lambda_{102}C_{w_y} - 2\alpha w_3C_{w_yw_z} \\
&-2\alpha w_2\lambda_{120}C_{w_y} - \beta w_4\lambda_{202}^* - \beta w_3\lambda_{201}C_{w_z} - \beta w_2\lambda_{220}^* - \beta w_1\lambda_{210}C_{w_x} < 0
\end{aligned} \quad ...(41)$$

From equations (19) and (33), $MSE(t_2) < MSE(t_{dc-2})$ if

$$\begin{aligned}
&w_4^2\lambda_{004}^* + C_{w_z}^2(w_3^2-1) + w_2^2\lambda_{040}^* + 2w_3w_4\lambda_{003}C_{w_z} + 2w_2w_3\lambda_{021}C_{w_z} + 2C_{w_xw_z}(w_1w_3-1) \\
&+2w_2w_4\lambda_{022}^* + 2w_1w_4\lambda_{012}C_{w_x} + 2w_1w_2\lambda_{030}C_{w_x} - 2\alpha w_2\lambda_{120}C_{w_y} - 2\alpha C_{w_yw_z}(w_3-1) \\
&+C_{w_x}^2(w_1^2-1) - 2\alpha w_4\lambda_{102}C_{w_y} - 2\alpha C_{w_yw_x}(w_1-1) - \beta\lambda_{201}C_{w_z}(w_3-1) - \beta w_2\lambda_{220}^* \\
&-\beta w_4\lambda_{202}^* - \beta\lambda_{210}C_{w_x}(w_1-1) < 0
\end{aligned} \quad ...(42)$$

From equations (10) and (38), $MSE(t_3) < MSE(t_0)$ if equation (41) is satisfied.

From equations (22) and (38), $MSE(t_3) < MSE(t_{dc-3})$ if equation (42) is satisfied.

RESULTS AND DISCUSSION

One artificial population and one real dataset are considered for numerical comparison of the proposed and existing estimators. The estimators are evaluated in terms of MSE, absolute relative bias (ARB) and percent relative efficiency (PRE). Chutiman and Kumphon (2008) generated a rare and clustered population for study variable y and two associated auxiliary variables x and z. For artificial population, simulated y, x and z values are studied from Chutiman and Kumphon (2008). To avoid zeros, artificial population is generated using the relation,

$$y_i = \begin{cases} 4yc_i & \text{if } yc_i > 0 \\ y_i \sim Poi(2) & \text{otherwise} \end{cases}$$

where yc_i are the simulated values from Chutiman and Kumphon (2008). Auxiliary variables are generated on the same pattern. The condition of interest is $C = \{y : y > 5\}$. The neighbourhood is defined as the four adjacent units. The study region is partitioned into 20 × 20 = 400 square units. The initial sample in phase one varies from 40 to 340 and in phase two, it varies from 5 to 170. The data statistics for simulated population are:

$N = 400, \overline{Y}_w = 8.9651, \overline{X}_w = 5.1625, \overline{Z}_w = 16.83, S^2_{w_y}$

$= 404.7388, S^2_{w_x} = 136.0429,$

$S^2_{w_z} = 2028.643, C_{w_y} = 2.244, C_{w_x} = 2.2593, C_{w_z}$

$= 2.6762, \rho_{w_y w_x} = 0.7574,$

$\rho_{w_x w_z} = 0.8806, \rho_{w_y w_z} = 0.8706, C_{w_y w_x} = 3.8401, C_{w_y w_z}$

$= 5.2286, C_{w_x w_z} = 5.3243,$

$\lambda_{400} = 17.3587, \lambda_{040} = 89.2648, \lambda_{004} = 26.4474, \lambda_{300}$

$= 3.4746, \lambda_{030} = 7.71,$

$\lambda_{003} = 4.2433, \lambda_{120} = 3.6728, \lambda_{102} = 3.4466, \lambda_{210}$

$= 2.7683, \lambda_{012} = 4.3969,$

$\lambda_{201} = 3.1124, \lambda_{021} = 5.4827, \lambda_{220} = 17.149, \lambda_{202}$

$= 17.6509, \lambda_{022} = 39.3448,$

For real dataset, counts of teals in 5000 km^2 area, which is partitioned into 200 25 km^2 quadrats in central Florida are studied from Smith *et al*. (1995). Data on blue winged teal and green winged teal are used as auxiliary variables x and z, respectively to generate the study variable y, using the model,

$$y_i = 4x_i + 4z_i + e_i$$

where $e_i \sim N(0, x_i + z_i)$. The condition of interest is $C = \{y : y > 0\}$. The initial sample in phase one varies from 20 to 150 and in phase two, it varies from 2 to 75. The data statistics for real dataset are:

$N = 200, \overline{Y}_w = 361.9401, \overline{X}_w = 70.6048, \overline{Z}_w = 12.01, S^2_{w_y}$

$= 3498192, S^2_{w_x} = 130872.4,$

$S^2_{w_z} = 12816.53, C_{w_y} = 5.1676, C_{w_x} = 5.1238, C_{w_z}$

$= 9.4263, \rho_{w_y w_x} = 0.9999,$

$\rho_{w_x w_z} = 0.5442, \rho_{w_y w_z} = 0.5443, C_{w_y w_x} = 26.4751, C_{w_x w_z}$

$= 26.5136,$

$\lambda_{400} = 26.4596, \lambda_{040} = 26.4355, \lambda_{004} = 96.9604, \lambda_{300}$

$= 5.0454, \lambda_{030} = 5.0418,$

$\lambda_{003} = 9.7831, \lambda_{120} = 5.0430, \lambda_{102} = 5.1769, \lambda_{210}$

$= 5.0442, \lambda_{012} = 5.1757,$

$\lambda_{201} = 2.7462, \lambda_{021} = 2.7450, \lambda_{220} = 26.4475, \lambda_{202}$

$= 27.1226, \lambda_{022} = 27.1103,$

The results for MSE, ARB and PRE of different estimators for estimation of population mean, variance and coefficient of variation are presented in Tables 1, 2 and 3, respectively. The estimators are proposed under three different situations, i.e. partial, no and full information about parameters of auxiliary variable. All the results reveal the same pattern for both artificial population and real dataset. The proposed estimators for population mean in three different situations are t_{1m}, t_{2m}, t_{3m}. Similarly estimators proposed for population variance and coefficient of variation are t_{1v}, t_{2v}, t_{3v} and t_{1c}, t_{2c}, t_{3c}, respectively. For small initial sample size, the proposed estimators are highly efficient with some amount of ARB present but as initial sample size increases, the proposed estimators are efficient with vanishing ARB. The proposed estimators for population mean, variance and coefficient of variation have high PRE (less MSE) than the existing estimators in all three situations. As initial sample size increases, PRE of proposed estimators for population mean, variance and coefficient of variation increases for situations 1 and 2 and remains constant for situation 3.

CONCLUSION

Exponential ratio type estimators are proposed for the general parameter ($\tau_{(\alpha,\beta)} = \overline{Y}^{\alpha}_w S^{\beta}_{w_y}$) using two auxiliary variables in adaptive cluster sampling under two phase sampling schemes. The results from both populations depict that the proposed estimators for general parameter are more efficient than the usual estimator proposed by Thompson (1990) and the ratio type estimator proposed by Dryver and Chao (2007) in all three situations. This shows that use of both population mean and variance of auxiliary variables is beneficial.

Table 1: *MSE*, *ARB* and *PRE* results of different estimators of population mean under adaptive cluster sampling

Simulated population								Real dataset							
							MSE								
n	t_{1m}	t_{dc-1m}	t_{2m}	t_{dc-2m}	t_{3m}	t_{dc-3m}	t_{0m}	n	t_{1m}	t_{dc-1m}	t_{2m}	t_{dc-2m}	t_{3m}	t_{dc-3m}	t_{0m}
5	15.871	446.07	22.852	147.14	15.513	155.78	79.932	2	97105	$2.126e^{7}$	$1.574e^{5}$	$5.368e^{6}$	27.033	$5.732e^{6}$	$1.731e^{6}$
10	7.9729	199.67	14.084	69.337	7.6584	76.902	39.46	5	75517	$7.717e^{6}$	$1.224e^{5}$	$1.975e^{6}$	10.649	$2.258e^{6}$	$6.821e^{5}$
20	3.9822	82.682	8.8522	31.436	3.731	37.465	19.224	10	61042	$3.329e^{6}$	99004	$8.714e^{5}$	5.1882	$1.100e^{6}$	$3.323e^{5}$
30	2.6082	49.74	6.2683	19.762	2.4193	24.293	12.465	15	50935	$1.962e^{6}$	82560	$5.237e^{5}$	3.3696	$7.145e^{6}$	$2.158e^{5}$
80	0.8477	16.224	2.025	6.4281	0.7855	7.8874	4.0472	20	43395	$1.324e^{6}$	69966	$3.594e^{5}$	2.4576	$5.211e^{6}$	$1.574e^{5}$
120	0.4815	10.765	0.9775	3.9603	0.4556	4.5747	2.3474	50	10901	$6.312e^{5}$	17491	$1.332e^{5}$	0.8192	$1.737e^{6}$	52473
170	0.2765	7.4985	0.4302	2.4897	0.2671	2.6817	1.3761	75	4302	$3.214e^{5}$	5597	83189	0.4533	96121	29035
							ARB								
n	t_{1m}	t_{dc-1m}	t_{2m}	t_{dc-2m}	t_{3m}	t_{dc-3m}	t_{0m}	n	t_{1m}	t_{dc-1m}	t_{2m}	t_{dc-2m}	t_{3m}	t_{dc-3m}	t_{0m}
5	2.61	4.06	1.69	1.49	1.9	1.68	0.0	2	1699	122.4	0.24	39.78	0.26	43.75	0.0
10	1.22	1.82	0.75	0.66	0.94	0.83	0.0	5	750	44.7	0.08	14.14	0.1	17.23	0.0
20	0.51	0.76	0.3	0.27	0.45	0.4	0.0	10	308	19.5	0.03	5.89	0.05	8.39	0.0
30	0.32	0.46	0.18	0.16	0.29	0.26	0.0	15	174	11.6	0.02	3.36	0.03	5.45	0.0
80	0.11	0.15	0.05	0.05	0.09	0.08	0.0	20	88	7.9	0.01	2.21	0.02	3.97	0.0
120	0.06	0.09	0.04	0.03	0.05	0.04	0.0	50	78	3.6	0.0054	0.88	0.0081	1.32	0.0
170	0.03	0.06	0.02	0.02	0.03	0.02	0.0	75	2.01	1.8	0.0036	0.59	0.0045	0.73	0.0
							PRE								
n	t_{1m}	t_{dc-1m}	t_{2m}	t_{dc-2m}	t_{3m}	t_{dc-3m}	t_{0m}	n	t_{1m}	t_{dc-1m}	t_{2m}	t_{dc-2m}	t_{3m}	t_{dc-3m}	t_{0m}
5	503.63	17.92	349.79	54.33	515.27	51.31	100	2	1783.3	8.14	1099.8	32.25	$6.405e^{6}$	30.21	100
10	494.93	19.76	280.18	56.91	515.27	51.31	100	5	903.31	8.84	557.1	34.53	$6.405e^{6}$	30.21	100
20	482.75	23.25	217.18	61.16	515.27	51.31	100	10	544.43	9.98	335.68	38.14	$6.405e^{6}$	30.21	100
30	477.93	25.06	198.87	63.08	515.27	51.31	100	15	423.84	10.99	261.43	41.21	$6.405e^{6}$	30.21	100
80	477.41	24.95	199.87	62.96	515.27	51.31	100	20	362.76	11.88	224.99	43.79	$6.405e^{6}$	30.21	100
120	487.55	21.8	240.16	59.27	515.27	51.31	100	50	481.32	8.31	300	39.36	$6.405e^{6}$	30.21	100
170	497.64	18.35	319.91	55.27	515.27	51.31	100	75	674.84	9.03	518.72	34.9	$6.405e^{6}$	30.21	100

Table 2: *MSE, ARB* and *PRE* results of different estimators of population variance under adaptive cluster sampling

Simulated population								Real dataset							
MSE															
n	t_{1v}	t_{dc-1v}	t_{2v}	t_{dc-2v}	t_{3v}	t_{dc-3v}	t_{0v}	n	t_{1v}	t_{dc-1v}	t_{2v}	t_{dc-2v}	t_{3v}	t_{dc-3v}	t_{0v}
5	90128	$7.586e^{5}$	$1.317e^{5}$	$3.500e^{5}$	80594	$3.269e^{5}$	$5.292e^{5}$	2	$8.656e^{12}$	$2.000e^{15}$	$1.402e^{13}$	$5.078e^{14}$	$5.440e^{8}$	$5.431e^{14}$	$1.542e^{14}$
10	48134	$3.481e^{5}$	84539	$1.815e^{5}$	39787	$1.614e^{5}$	$2.612e^{5}$	5	$6.732e^{12}$	$7.258e^{14}$	$1.090e^{13}$	$1.864e^{14}$	$2.143e^{8}$	$2.139e^{14}$	$6.075e^{13}$
20	26036	$1.509e^{5}$	55049	94717	19383	78637	$1.272e^{5}$	10	$5.443e^{12}$	$3.130e^{14}$	$8.817e^{12}$	$8.201e^{13}$	$1.044e^{8}$	$1.042e^{14}$	$2.959e^{13}$
30	17568	92887	39374	63076	12568	50989	82537	15	$4.542e^{12}$	$1.845e^{14}$	$7.352e^{12}$	$4.916e^{13}$	$6.781e^{7}$	$6.770e^{13}$	$1.922e^{13}$
80	5717	30253	12713	20447	4080	16555	26797	20	$3.869e^{12}$	$1.245e^{14}$	$6.231e^{12}$	$3.366e^{13}$	$4.945e^{7}$	$4.938e^{13}$	$1.402e^{13}$
120	3044	19320	6001	11240	2366	9602	15542	50	$9.881e^{11}$	$5.934e^{13}$	$1.557e^{12}$	$1.253e^{13}$	$1.648e^{7}$	$1.646e^{13}$	$4.673e^{12}$
170	1601	12794	2523	6140	1387	5628	9111	75	$4.033e^{11}$	$3.023e^{13}$	$4.985e^{11}$	$7.850e^{12}$	$9.122e^{6}$	$9.107e^{12}$	$2.585e^{12}$
ARB															
n	t_{1v}	t_{dc-1v}	t_{2v}	t_{dc-2v}	t_{3v}	t_{dc-3v}	t_{0v}	n	t_{1v}	t_{dc-1v}	t_{2v}	t_{dc-2v}	t_{3v}	t_{dc-3v}	t_{0v}
5	9.21	2.48	10.96	0.52	12.37	0.59	0.0	2	1657.7	123.3	5.55	40.35	6.11	44.38	0.0
10	4.09	1.09	4.87	0.23	6.11	0.29	0.0	5	683.5	45	1.97	14.34	2.41	17.48	0.0
20	1.64	0.43	1.99	0.09	2.97	0.14	0.0	10	299.3	19.6	0.82	5.98	1.17	8.51	0.0
30	0.98	0.25	1.19	0.05	1.93	0.09	0.0	15	160.5	11.6	0.47	3.42	0.76	5.53	0.0
80	0.34	0.08	0.38	0.018	0.62	0.03	0.0	20	83.5	7.9	0.31	2.24	0.55	4.03	0.0
120	0.21	0.05	0.26	0.012	0.36	0.017	0.0	50	80.8	3.7	0.12	0.89	0.18	1.34	0.0
170	0.16	0.04	0.18	0.008	0.21	0.01	0.0	75	4.99	1.9	0.08	0.6	0.1	0.74	0.0
PRE															
n	t_{1v}	t_{dc-1v}	t_{2v}	t_{dc-2v}	t_{3v}	t_{dc-3v}	t_{0v}	n	t_{1v}	t_{dc-1v}	t_{2v}	t_{dc-2v}	t_{3v}	t_{dc-3v}	t_{0v}
5	587.22	69.76	401.84	151.21	656.69	161.87	100	2	1781.5	7.71	1099.9	30.36	$2.834e^{7}$	28.39	100
10	542.8	75.04	309.06	143.88	656.69	161.87	100	5	902.41	8.37	557.13	32.58	$2.834e^{7}$	28.39	100
20	488.89	84.35	231.23	134.39	656.69	161.87	100	10	543.76	9.45	335.69	36.09	$2.834e^{7}$	28.39	100
30	469.8	88.86	209.62	130.85	656.69	161.87	100	15	423.19	10.42	261.44	39.1	$2.834e^{7}$	28.39	100
80	468.66	88.57	210.79	131.06	656.69	161.87	100	20	362.31	11.26	224.99	41.64	$2.834e^{7}$	28.39	100
120	510.46	80.45	258.98	138.27	656.69	161.87	100	50	476.3	7.87	300	37.29	$2.834e^{7}$	28.39	100
170	568.83	71.21	361.08	148.37	656.69	161.87	100	75	641.12	8.55	518.74	32.94	$2.834e^{7}$	28.39	100

Table 3: *MSE*, *ARB* and *PRE* results of different estimators of population coefficient of variation under adaptive cluster sampling

Simulated population								Real dataset							
MSE															
n	t_{1c}	t_{dc-1c}	t_{2c}	t_{dc-2c}	t_{3c}	t_{dc-3c}	t_{0c}	n	t_{1c}	t_{dc-1c}	t_{2c}	t_{dc-2c}	t_{3c}	t_{dc-3c}	t_{0c}
5	0.2487	55.893	0.3457	24.646	0.2203	27.645	1.321	2	5.1846	7059.8	8.412	2759.3	0.005	3025.9	92.481
10	0.1337	25.367	0.2185	11.022	0.1087	13.647	0.6522	5	4.0292	2572.6	6.5407	984.6	0.002	1192.1	36.432
20	0.0728	10.777	0.1405	4.5563	0.053	6.649	0.3177	10	3.2573	1117.5	5.288	413	0.001	580.7	17.75
30	0.0493	6.5688	0.1001	2.7385	0.0343	4.3113	0.206	15	2.7166	662.7	4.4096	237.3	0.0006	377.2	11.527
80	0.016	2.1408	0.0323	0.8932	0.0111	1.3998	0.0669	20	2.316	449.7	3.7369	156.6	0.0005	275.1	8.4074
120	0.0085	1.3862	0.0154	0.5986	0.0065	0.8119	0.0388	50	0.5906	205.3	0.9343	62.1	0.0002	91.7	2.8025
170	0.0044	0.9449	0.0066	0.4093	0.0038	0.4759	0.0227	75	0.1888	107.4	0.299	41.3	8E-05	50.7	1.5507
ARB															
n	t_{1c}	t_{dc-1c}	t_{2c}	t_{dc-2c}	t_{3c}	t_{dc-3c}	t_{0c}	n	t_{1c}	t_{dc-1c}	t_{2c}	t_{dc-2c}	t_{3c}	t_{dc-3c}	t_{0c}
5	1.49	7.01	2.01	3.21	2.24	3.64	0.17	2	443.3	183.5	5.01	81.02	6.03	88.61	5.19
10	0.69	3.19	0.9	1.41	1.1	1.79	0.08	5	193.2	67.4	1.58	29.01	2.37	34.91	2.04
20	0.31	1.36	0.37	0.57	0.54	0.87	0.04	10	79.9	29.6	0.52	12.23	1.16	17.01	0.99
30	0.19	0.83	0.22	0.34	0.35	0.56	0.02	15	47.6	17.7	0.22	7.06	0.75	11.04	0.65
80	0.06	0.27	0.07	0.11	0.11	0.18	0.009	20	23.3	12.2	0.09	4.68	0.55	8.06	0.47
120	0.03	0.17	0.04	0.07	0.06	0.1	0.005	50	4.6	5.4	0.07	1.84	0.18	2.68	0.15
170	0.02	0.11	0.03	0.05	0.003	0.47	0.003	75	2.8	2.8	0.06	1.22	0.1	1.48	0.09
PRE															
n	t_{1c}	t_{dc-1c}	t_{2c}	t_{dc-2c}	t_{3c}	t_{dc-3c}	t_{0c}	n	t_{1c}	t_{dc-1c}	t_{2c}	t_{dc-2c}	t_{3c}	t_{dc-3c}	t_{0c}
5	531.03	2.36	382.15	5.36	599.71	4.78	100	2	1783.8	1.31	1099.4	3.35	$1.846e^{6}$	3.06	100
10	487.81	2.57	298.41	5.92	599.71	4.78	100	5	904.2	1.42	557.01	3.7	$1.846e^{6}$	3.06	100
20	436.08	2.95	226.16	6.97	599.71	4.78	100	10	544.8	1.59	335.65	4.3	$1.846e^{6}$	3.06	100
30	417.94	3.14	205.77	7.52	599.71	4.78	100	15	424.3	1.74	261.42	4.85	$1.846e^{6}$	3.06	100
80	418.85	3.12	206.88	7.48	599.71	4.78	100	20	363	1.87	244.98	5.37	$1.846e^{6}$	3.06	100
120	456.64	2.8	252.14	6.48	599.71	4.78	100	50	474.5	1.36	299.97	4.51	$1.846e^{6}$	3.06	100
170	513.52	2.41	345.68	5.56	599.71	4.78	100	75	821.5	1.44	518.63	3.76	$1.846e^{6}$	3.06	100

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