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RESEARCH ARTICLE

An efficient algorithm to calculate relative permittivity from multi-layered stripline based measurements at microwave frequencies

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Abstract: In recent years, multi-layer stripline or microstrip line based dielectric measurement has drawn the attention of many researchers. These methods are particularly attractive as they can readily be used for the dielectric measurement of non-metal clad material. Measurement setups developed using the multi-layer approach have the added advantage that they need very little sample preparation. However, the extraction of dielectric properties in these experimental setups is complex due to the multi-layer nature of the problem. At present, this is usually done using brute-force approaches, which take long computational times and large memory. This paper presents an efficient technique based on the Newton Raphson algorithm to address this. The algorithm developed is coded in Matlab and the results show a significant improvement over the brute-force method.

Keywords: Dielectric measurement, multi-layer, Newton Raphson, permittivity.

INTRODUCTION

It is well known that at microwave frequencies, the permittivity and loss-tangent of dielectric material in the form of sheets can be obtained by fabricating a resonant circuit and then measuring the resonance frequency and scattering parameters. However, if the material under test cannot be metal cladded, fabricating the resonant circuit poses problems. This is usually circumvented by using a high quality printed circuit board (PCB) as the substrate and then making the measurements with the test sample placed on the top as a superstrate. Since a PCB is used as the substrate, fabrication of the resonant circuit does not pose any difficulties.

The focus of this paper is on the measurement of the permittivity. The measured resonance frequency gives the effective relative dielectric constant (ε_{reff}), that is the permittivity of the multi-layer dielectric composite. The effective relative dielectric constant is related to the dielectric constants of the test sample and the substrate *via* complex equations due to the multi-layer nature of the problem. Thus, given the effective dielectric constant (from measurements), obtaining the dielectric constant of the test sample is not straightforward. In literature (Bernard & Gautray, 1991; Suzuki & Hotchi, 2008), the solution is obtained by constructing a graph of ε_{reff} against each possible value of the relative permittivity of the test sample within a specified range. For example in Suzuki and Hotchi (2008), ε_{reff} is calculated for relative permittivity of the test sample values ranging from 1 to 1000 in steps of 0.0001. The unknown relative permittivity (of the test sample) corresponding to the measured ε_{reff} is obtained from the graph. In this paper, a numerical solution to the problem is proposed using the Newton Raphson algorithm. Compared to the brute-force approach adopted at present, the proposed method gives a significant improvement in time taken to extract the relative permittivity of the test sample from ε_{reff}. An exact comparison is provided in the section on results and discussion.

Multi-layer striplines and microstrip lines can be analysed using several methods. These different approaches can be classified as methods based on Green's function techniques (Yamashita, 1968; Bhat & Koul, 1990), conformal mappings (Bhat & Koul, 1990;

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Svacina, 1992), and hybrid-mode analysis (Kitazawa, 1989; Bhat & Koul, 1990). A dielectric measurement method based on the Green's functions is described by Bernard and Gautray (1991). A method based on hybrid-mode analysis is described by Suzuki and Hotchi (2008), which has later been adopted in a commercial product (Keysight Technologies & KEYCOM corp., 2014). Methods based on conformal mapping techniques are described by Rashidian *et al.* (2012) and Samarasinghe *et al.* (2016). Irrespective of the differences in the methods of analysis, multi-layer stripline or microstrip line based measurement techniques using these methods share the advantages of easy sample preparation (which does not require any special preparation other than cutting a sample to a specified size) and applicability for the measurement of non-metal clad samples.

The numerical technique developed in this paper is for a stripline based measurement formulated using Green's functions. In the section on methodology, the analysis of a multi-layer strip line using the Green's function technique is reviewed. Moreover, a numerical solution based on the Newton Raphson algorithm is also developed in the same section. Results to confirm the superior performance of the proposed method are presented in the subsequent section.

METHODOLOGY

This paper introduces a computationally efficient method to extract relative permittivity from multilayer stripline based measurement setups. Therefore, in the proceeding subsection, we review how the equations used in the calculations are obtained. Then we formulate the problem using Newton Raphson algorithm, which is the main contribution of the paper.

Analysis of the multi-layer stripline

The method used in this paper and in the paper by Bernard and Gautray (1991) is based on the formulae originally derived in Yamashita (1968) for the analysis of inhomogeneous transmission lines. The cross-section of the transmission line considered in Yamashita (1968) is shown in Figure 1. Here the figure is drawn upside down compared to the original publication (Yamashita, 1968) to fit into the requirements of this paper. In Yamashita (1968), it is shown that the capacitance per unit length (C) of the transmission line is given by:

$$\frac{1}{C} = \frac{1}{\pi Q^2}\int_0^{+\infty} \hat{f}(\beta)\hat{\phi}(\beta,d)\,d\beta \qquad ...(1)$$

where $\hat{f}(\beta)$ is the Fourier transform of the charge distribution $f(x)$, Q is the total charge on the centre conductor, that is,

$$Q = \int_{-w_0/2}^{+w_0/2} f(x)\,dx \qquad ...(2)$$

and, $\hat{\phi}(\beta,d)$ is the Fourier transform of the potential distribution $\phi(x,y)$ at $y = d$. Furthermore, $\hat{\phi}(\beta,d)$ is given by Yamashita (1968):

$$\hat{\phi}(\beta,d) = \frac{1}{\varepsilon}\hat{f}(\beta)\hat{g}(\beta) \qquad ...(3)$$

where $\hat{g}(\beta)$ is the Fourier transform with respect to x of the Green's function of the governing Poisson's equation. $\hat{g}(\beta)$ is given by equation (10) of Yamashita (1968) where ε_1, ε_2, and ε_3 are the permittivities (absolute) of the respective layers (Figure 1).

$$\hat{g}(\beta) = \frac{\varepsilon_1\coth(|\beta|h) + \varepsilon_2\coth(|\beta|s)}{|\beta|\{\varepsilon_1\coth(|\beta|h)[\varepsilon_3\coth(|\beta|d) + \varepsilon_2\coth(|\beta|s)] + \varepsilon_2[\varepsilon_2 + \varepsilon_3\coth(|\beta|d)\coth(|\beta|s)]\}} \qquad ...(4)$$

β is the special frequency with respect to x. Equations (1) through (4) can be used to solve for cases involving stripline, microstrip line, and the microstrip line with a top metal cover. Equations corresponding to a stripline can be obtained by setting $h = 0$ in equation (4). Similarly, by setting $h = \infty$ and $\varepsilon_1 = \varepsilon_0$, equations corresponding to an unshielded microstrip line with a superstrate (test sample) can be obtained. In this paper, the emphasis is on stripline based measurement techniques such as those described in Bernard and Gautray (1991). Therefore, the equations derived and the algorithm developed next is for the case of striplines. However, these results can be easily extended to a microstrip based measurement setup by considering the case where $h = \infty$ and $\varepsilon_1 = \varepsilon_0$.

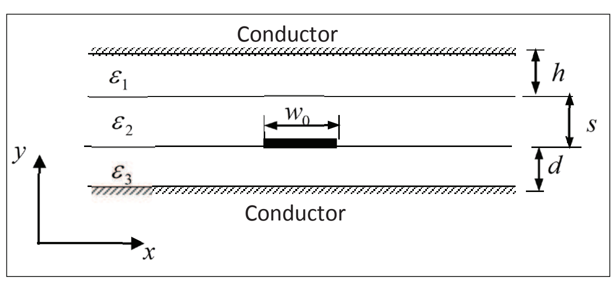

Figure 1: Shielded double layer microstrip line

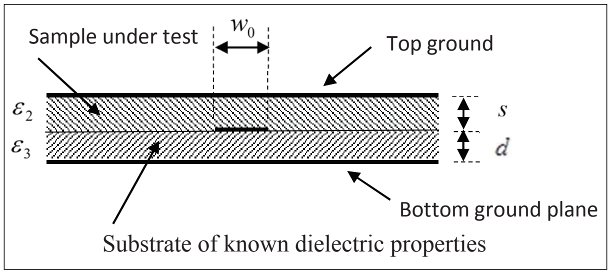

Figure 2: Dielectric stack

The cross-section of a stripline based measurement setup is shown in Figure 2. Setting $h = 0$ in equation (4) and substituting the result in equation (1) gives:

$$\frac{1}{C} = \frac{1}{\pi Q^2}\int_0^\infty \frac{[\hat{f}(\beta)]^2}{\beta[\varepsilon_3 \coth \beta d + \varepsilon_2 \coth \beta s]}.d\beta . \quad ...(5)$$

In deriving equations (1) to (5), for simplicity it is assumed that the thickness of the centre conductor is negligible. However, this assumption can be relaxed by adding one extra term to the kernel of the integral of equation (1) (Yamashita, 1968). Derivations also assume that the mode of propagation is quasi-TEM. This condition can easily be satisfied by carrying out the measurements below the cut-off frequency of higher order modes. Moreover, since the propagating mode is quasi-TEM, the capacitance per unit length of the transmission line can be expressed as,

$$C = \frac{C_0}{\varepsilon_{reff}} \quad ...(6)$$

where ε_{reff} is the effective relative permittivity and C_0 is the capacitance of the transmission line if the substrate and the test material are replaced by air. Therefore, the effective relative permittivity is given by:

$$\varepsilon_{reff}(\varepsilon_{r2}) = \frac{\int_0^\infty \frac{[\hat{f}(\beta)]^2 d\beta}{\beta[\coth(\beta d) + \coth(\beta s)]}}{\int_0^\infty \frac{[\hat{f}(\beta)]^2 d\beta}{\beta[\varepsilon_{r3}\coth(\beta d) + \varepsilon_{r2}\coth(\beta s)]}} . \quad ...(7)$$

Note that in equation (7), ε_{r2} and ε_{r3} are the relative permittivities corresponding to the absolute permittivities ε_2 and ε_3, respectively as depicted in Figure 1. In dielectric measurement methods based on stripline fixtures (Bernard & Gautray, 1991; Suzuki & Hotchi, 2008) the relative permittivity of the test sample ε_{r2}, is obtained from ε_{reff}, which is the measured quantity. However, this is not straightforward as ε_{r2} and ε_{reff} are connected through a complicated equation as shown in equation (7). In literature, relative permittivity of the test sample is obtained by constructing a graph of ε_{reff} *vs* ε_{r2}, which requires equation (7) to be evaluated at a large number of points. In the next section, we introduce a numerical technique based on Newton Raphson method to solve equation (7) more efficiently.

Foregoing work requires a trial function to be selected for $f(x)$, the charge distribution. Following Yamashita (1968), $f(x)$ is selected as:

$$f(x) = \begin{cases} 1 + \left|\frac{2x}{w_0}\right|^3 & for\ |x| \le w_0/2 \\ 0 & otherwise. \end{cases} \quad ...(8)$$

which gives (Yamashita, 1968),

$$\frac{\hat{f}(\beta)}{Q} = \frac{8}{5}\left\{\frac{\sin(\beta w_0/2)}{\beta w_0/2}\right\} + \frac{12}{5(\beta w_0/2)^2} .$$

$$\left\{\cos(\beta w_0/2) - \frac{2\sin(\beta w_0/2)}{\beta w_0/2} + \frac{\sin^2(\beta w_0/4)}{(\beta w_0/4)^2}\right\}. \quad ...(9)$$

Newton Raphson formulation

In the Newton Raphson method, the solutions of the equation $G(x) = 0$ is obtained using the iterative formula:

$$x_{n+1} = x_n - \frac{G(x_n)}{G'(x_n)} \quad ...(10)$$

where $G'(x)$ is the derivative of $G(x)$ with respect to x. Therefore, if we define a function of ε_{r2} as,

$$F(\varepsilon_{r2}) = \varepsilon_{reffM} - \varepsilon_{reff}(\varepsilon_{r2}), \quad ...(11)$$

where ε_{reffM} is the measured effective permittivity, the solution of equation (11) will give the relative permittivity of the test sample, ε_{r2}. With little manipulation, it can be shown that the derivative of $F(\varepsilon_{r2})$ with respect to ε_{r2} is given by,

$$F'(\varepsilon_{r2}) = \frac{C_0[\varepsilon_{reff}(\varepsilon_{r2})]^2}{\pi\varepsilon_0 Q^2}\int_0^\infty \frac{\coth(\beta s).[\hat{f}(\beta)]^2}{\beta[\varepsilon_{r3}\coth(\beta d) + \varepsilon_{r2}\coth(\beta s)]^2} d\beta . \quad ...(12)$$

In the practical implementation of the algorithm, both $F(\varepsilon_{r2})$ and $F'(\varepsilon_{r2})$ are evaluated using numerical integration. The steps of the algorithm can be summarised as shown in Figure 3. In the algorithm, ε_{r2}^{n} is the value of ε_{r2} after the n^{th} iteration.

Step 1:

Set ε_{r2}^{0} to an initial guess.

Set iteration number n to zero.

Step 2:

Calculate $F(\varepsilon_{r2}^{n})$ and $F'(\varepsilon_{r2}^{n})$ using equations (10) and (11).

Find ε_{r2}^{n+1} using the equation

$$\varepsilon_{r2}^{n+1} = \varepsilon_{r2}^{n} - \frac{F(\varepsilon_{r2}^{n})}{F'(\varepsilon_{r2}^{n})}$$

Step 3:

Check if $\left|\varepsilon_{r2}^{n+1} - \varepsilon_{r2}^{n}\right| < \eta$ where η is a specified accuracy.

If the check fails, increase n by 1 and go back to Step 2.

Take ε_{r2}^{n+1} as the relative permittivity of the sample otherwise.

Figure 3: Algorithm

RESULTS AND DISCUSSION

The performance of the proposed algorithm is demonstrated by considering a measurement scenario with parameters given in Table 1. The result shown in Figure 6 was obtained using equation (7). The equation was coded using MATLAB v14b. The relative permittivity of the test sample, ε_{r2} varied from 1 to 70 to cover most practically available material. This result is equivalent to Figure 3 of Bernard and Gautray (1991), the graph used to read the relative permittivity of the test sample from the effective permittivity. Experimental verification of the results shown in Figure 6 is not necessary as it has been done in Yamashita (1968). However, the results shown in Figure 6 were cross checked against the results obtained using a finite element electromagnetic simulation to rule out any coding errors. The problem was simulated using Ansoft high frequency electromagnetic field simulation (HFSS V10.1). In the HFSS, a transmission line of specified dimensions sandwiched between the substrate and the test sample was considered (Figure 4). The line width, w_0 was selected deliberately to mismatch with the 50 Ω port impedance so that the resonance due to reflections from ports will be prominent. Figure 5 shows the response of S11. The effective permittivity, ε_{reff} , can be calculated from the result:

$$f_0 = \frac{nc}{2l\sqrt{\varepsilon_{reff}}} \quad ...(13)$$

where f_0 is the resonance frequency, c is the speed of propagation of electromagnetic waves in free space, l is the line length and n is an integer. The effective permittivity values obtained from HFSS are marked as crosses in Figure 6, which illustrates the accuracy of coding and the validity of the theory presented in Yamashita (1968). It should be noted that although Figure 6 resembles a straight line, it is not a straight line as it is evident from the gradient of the effective relative permittivity plotted in Figure 7.

Table 1: Parameters used

Substrate parameters	Relative permittivity	3.2
	Thickness	0.762 mm (30 mil)
	Line width (w_0)	1.8 mm
	Line length (l)	76.2 mm (3 in)
Test sample	Thickness	2 mm

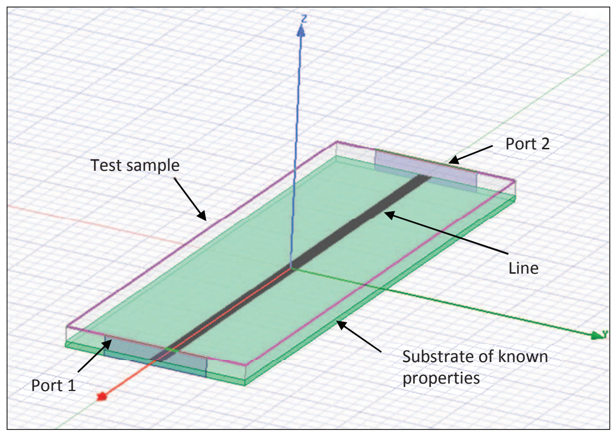

Figure 4: HFSS simulation setup

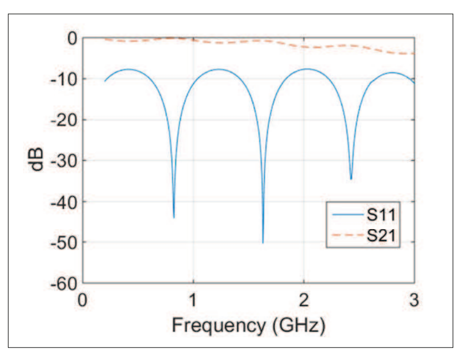

Figure 5: Response of S11 and S22, $\varepsilon_{r2} = 10$

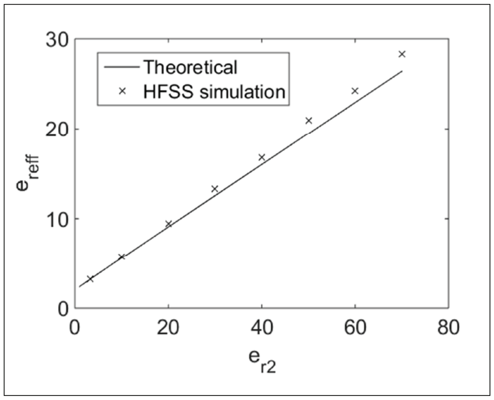

Figure 6: Variation of ε_{reff} *vs.* ε_{r2}

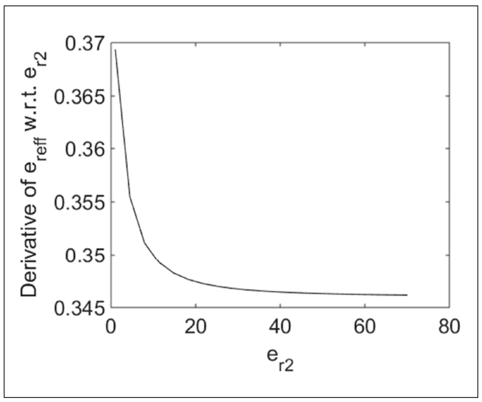

Figure 7: Variation of the derivative of ε_{reff}

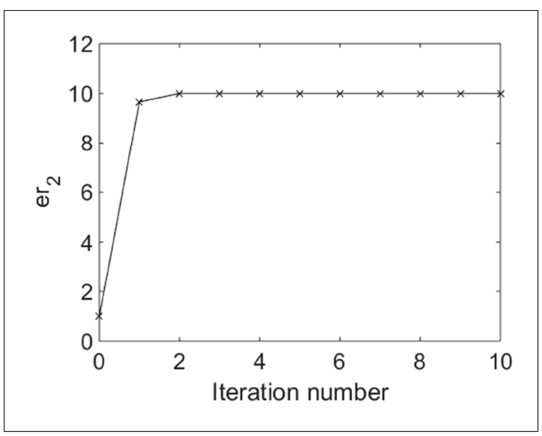

Figure 8: Convergence characteristics

A test sample of ε_{r2} = 10.0 is considered to demonstrate the performance of the algorithm. Test setup and sample parameters are as specified in Table 1. Using equation (7), it can be shown that ε_{reff} in this case is 5.6. In a measurement scenario, what is measured is ε_{reff} and the proposed algorithm is used to find ε_{r2}. Figure 8 shows how the algorithm converges to the solution when the measured value (ie., ε_{reff}) is specified as 5.6. The Newton Raphson algorithm was initialised using an initial guess of $\varepsilon_{r2} = 1.0$. The results clearly demonstrate that the algorithm converges to within 1 % of the true value in less than 3 iterations. This should be expected as the variation of ε_{reff} with ε_{r2} is nearly linear. On the other hand, if we consider the brute-force approach (Bernard & Gautray, 1991; Suzuki & Hotchi, 2008), reading the permittivity with a 1 % accuracy (when ε_{r2} is 10.0) requires the computation of ε_{reff} for all values of ε_{r2} starting from 1 to 70 (the upper limit selected in this paper) in steps of 0.1. This requires the computation of equation (7) 691 times as opposed to 3 iterations of the Newton Raphson algorithm.

Commercial measurement solutions based on multi-layer stacks are relatively new to the industry (Keysight Technologies & KEYCOM corp., 2014) and it is envisaged that the method developed in this study will contribute to further enhance the efficiency of these products. The technique described also has many local applications. For example, Samarasinghe *et al.* (2016) describes a method of condition monitoring of transformers used in power distribution based on multi-layer stack measurements.

The method introduced in this paper is for multi-layer striplines and is based on the Newton-Raphson algorithm. It is possible to extend this work to cover other multi-layer technologies such as multi-layer microstrip lines and also other numerical algorithms used for the numerical solution of equations.

CONCLUSION

Multi-layer stripline/microstrip line based techniques are particularly attractive as they can be used with non-metal clad test samples and need very little effort in sample preparation (only required to cut the sample to a specified size). In this paper, it is demonstrated that the computational time involved in measurements can be dramatically reduced by employing numerical techniques (namely the Newton Raphson algorithm) to extract desired parameters. This is particularly important in automated measurement applications where averaging and narrow I.F. bandwidths are employed to reduce measurement noise. In addition, the reduction in computational time is also beneficial in applications where measurements are done over wide bandwidths.

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