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RESEARCH ARTICLE

Back-bending phenomena in even-even $^{110\text{-}118}$Te isotopes

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Abstract: In the present study the back-bending phenomena of moment of inertia are described at high-spin yrast bands of even-even $^{110\text{-}118}$Te isotopes. The Fermi energy, square of rotational frequency and the moment of inertia of the nuclei have been calculated from available experimental data. An interesting nuclear feature emerging from this study concerns the evolution of the moment of inertia, and the yrast line indicates about the nuclear shape. Fermi energy and the moment of inertia as a function of even neutron number of $^{110\text{-}118}$Te isotopes have been studied systematically. The moment of inertia as a function of the square of the rotational energy for even neutrons N = 58 to 66 in Te isotopes indicates the nature of back-bending properties. The investigation of back-bending phenomena in ordinary space for even-even Te isotopes with even neutron N = 58 to 66 are carried out and compared with gauge space.

Keywords: Back-bending, even $^{110\text{-}118}$Te isotopes, Fermi energy, moment of inertia, yrast band.

INTRODUCTION

In quantum electrodynamics the conservation of charge is a consequence of the so-called gauge invariance of the Hamiltonian. The conservation of a particle number is related to the rotational invariance in gauge space. The existence of a strong pair field, which introduces a deformation in gauge space breaks the rotational invariance, and as a result, a rotational motion in gauge space becomes possible. It is very important to consider analogies between the quantities in gauge space and in ordinary space (Bohr & Mottelson, 1975). In both spaces rotational and vibrational bands have been observed. One can thus study irregularities in rotational bands in 'back bending plots', similar to those in ordinary space, and a back bending or unbending behaviour may correspond to a shape transition.

It is known that low-lying collective quadrupole *E2* excitations occur in even-even Te (Z = 52), which have been studied both theoretically and experimentally (Bohr & Mottelson, 1975; Robinson *et al*., 1983; ZhongZe *et al*., 2000). The electric quadrupole moments of even $^{120\text{-}128}$Te isotopes have been studied within the framework of the semi-microscopic model (Lopac, 1970), two-proton core coupling model (Degrieck & Berghe, 1974), dynamic deformation model (Subber *et al*., 1987) and the interacting boson model-2 (IBM-2) (Sambataro, 1982; Rikovska *et al*., 1987).

The back-bending phenomena at high spins of even-even rare earth nuclei in the ground state rotational bands have been studied extensively in many nuclei (Najim & Kheder, 2013). A sudden decrease of the rotational frequency along with an anomalous increase in the moment of inertia has been found to occur in many deformed nuclei. The word back-bending refers to the phenomenon where a plot of twice the moment of inertia versus the square of rotational frequency for various spin states has an S-shaped form (Sirag, 2006). The variable moment of inertia (VMI) model has a term of two parameters added to the rotational energy equation by Mariscottoi *et al*. (1969) for the fitting of energies with the measured energies.

Recently, back-bending calculations were made using the angular momentum projected Tomm-Dancoff approximation (Sun & Egido, 1994), projected shell model (Wen-Hua & Jian-Zhong, 2010) and the projected

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configuration interaction (PCI) method in which the deformed intrinsic states are directly associated with shell model wave function (Gao *et al*., 2011). We have studied the evolution properties and the back-bending properties of yrast states for even-even $^{100-110}$Pd and $^{104-122}$Cd isotopes (Ahmed *et al.*, 2012; Hossain *et al*., 2013). Electromagnetic reduced transition probabilities of even-even $^{104-112}$Cd (Abdullah *et al*., 2013), $^{102-106}$Pd (Hossain *et al.*, 2014a) and $^{108-112}$Pd (Hossain *et al.*, 2014b) isotopes have been studied by IBM-1. So far the back-bending properties of yrast states of even-even $^{110-118}$Te isotopes have not been reported in the literature. Therfore, in this study an extensive analysis of the back-bending properties of even $^{110-118}$Te isotopes has been carried out.

Table 1: Excitation energies, moment of inertia and square of rotational frequency for even $^{110-118}$Te isotopes

Nuclei	I	$I(I+1)$	Transition level	E_y MeV	$2\vartheta/\hbar^2$ MeV^{-1}	$(\hbar\omega)^2$ (MeV)^2
^{110}Te	2	6	$2^+ \rightarrow 0^+$	0.657	9.132	0.1079
	4	20	$4^+ \rightarrow 2^+$	0.744	18.817	0.1384
	6	42	$6^+ \rightarrow 4^+$	0.824	26.699	0.1697
	8	72	$8^+ \rightarrow 6^+$	1.062	28.249	0.2819
	10	110	$10^+ \rightarrow 8^+$	1.368	27.778	0.4679
	12	156	$12^+ \rightarrow 10^+$	0.533	86.3	0.0710
	14	210	$14^+ \rightarrow 12^+$	0.688	78.488	0.1183
	16	272	$16^+ \rightarrow 12^+$	0.763	81.258	0.1455
^{112}Te	2	6	$2^+ \rightarrow 0^+$	0.689	8.708	0.1186
	4	20	$4^+ \rightarrow 2^+$	0.787	17.789	0.1548
	6	42	$6^+ \rightarrow 4^+$	0.821	26.797	0.1685
	8	72	$8^+ \rightarrow 6^+$	1.065	28.169	0.2835
	10	110	$10^+ \rightarrow 8^+$	0.864	43.981	0.1866
	12	156	$12^+ \rightarrow 10^+$	0.601	76.539	0.0903
	14	210	$14^+ \rightarrow 12^+$	0.713	75.736	0.1271
	16	272	$16^+ \rightarrow 14^+$	0.754	82.228	0.1421
	18	342	$18^+ \rightarrow 16^+$	0.957	73.145	0.2289
^{114}Te	2	6	$2^+ \rightarrow 0^+$	0.708	8.475	0.1253
	4	20	$4^+ \rightarrow 2^+$	0.775	18.065	0.1502
	6	42	$6^+ \rightarrow 4^+$	0.733	30.036	0.1343
	8	72	$8^+ \rightarrow 6^+$	0.871	34.443	0.1896
	10	110	$10^+ \rightarrow 8^+$	0.831	48.135	0.1726
	12	156	$12^+ \rightarrow 10^+$	0.596	77.181	0.0888
	14	210	$14^+ \rightarrow 12^+$	0.737	73.27	0.1357
	16	272	$16^+ \rightarrow 14^+$	0.691	89.725	0.1193
	18	342	$18^+ \rightarrow 16^+$	0.976	71.72	0.238
^{116}Te	2	6	$2^+ \rightarrow 0^+$	0.678	8.849	0.1149
	4	20	$4^+ \rightarrow 2^+$	0.680	20.588	0.1156
	6	42	$6^+ \rightarrow 4^+$	0.643	34.214	0.1034
	8	72	$8^+ \rightarrow 6^+$	0.770	38.961	0.1482
	10	110	$10^+ \rightarrow 8^+$	0.810	46.914	0.1640
	12	156	$12^+ \rightarrow 10^+$	0.764	60.209	0.1459
	14	210	$14^+ \rightarrow 12^+$	0.770	70.129	0.1482
^{118}Te	2	6	$2^+ \rightarrow 0^+$	0.605	9.917	0.915
	4	20	$4^+ \rightarrow 2^+$	0.600	23.33	0.090
	6	42	$6^+ \rightarrow 4^+$	0.614	35.831	0.0942
	8	72	$8^+ \rightarrow 6^+$	0.753	39.841	0.1417
	10	110	$10^+ \rightarrow 8^+$	0.786	48.346	0.1544
	12	156	$12^+ \rightarrow 10^+$	0.859	53.551	0.1844
	14	210	$14^+ \rightarrow 12^+$	0.903	59.800	0.2038

THEORETICAL CALCULATION

Moment of inertia (ϑ) and gamma energy E_γ

The relationship between the moment of inertia (ϑ) and gamma energy Eγ (Ahmed *et al*., 2012) is given by

$$2\vartheta/\hbar^2 = \frac{4I-2}{E(I)-E(I-2)} = \frac{4I-2}{E_\gamma} \quad ...(1)$$

And the relationship between Eγ and ħω (Scholten *et al*., 1978) is given by

$$\hbar\omega = \frac{E(I)-E(I-2)}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}} = \frac{E_\gamma}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}} \quad ...(2)$$

Fermi energy (Gauge space)

The Fermi energies are calculated from the following relationship (Hussien, 1993):

$$\lambda(N,I) = \frac{1}{2}[E_x(N+1,I) - E_x(N-1,I) - S_{2n}^{N+1}] \quad ...(3)$$

where N is the neutron number and S_{2n}^{N+1} is the separation energy.

$$S_{2n}^{N+1} = E_B\,(Z,N) - E_B\,(Z,\,N\text{-}2) \quad ...(4)$$

RESULTS AND DISCUSSION

The gamma ray energies, moment of inertia and square of rotational energy for the ground state band of even-even $^{110\text{-}118}$Te isotopes are presented in Table 1. The separation energy and the Fermi energy of even-even $^{112\text{-}118}$Te isotopes are presented in Tables 2 and 3.

Moment of inertia

The positive parity yrast levels are connected by a sequence of stretched E2 transition energies, which increases smoothly except around the back-bending region. The transition energy $\Delta E_{I,\,I\text{-}2}$ should increase linearly with I for the constant rotor as $\Delta E_{I,\,I\text{-}2} = I/2\vartheta$ (4I-2). However it does not increase, but decreases for certain I values. The moment of inertia $2\vartheta/\hbar^2$ and the rotational frequency $\hbar\omega$ have been calculated from the equations (1) and (2), respectively. The E_γ between E(I) and E(I-2) levels are obtained from ^{110}Te (Gürdal & Kondev, 2012), ^{112}Te (De Frenne & Jacobs, 1996), ^{114}Te (Blachot, 2012), ^{116}Te (Blachot, 2001) and ^{118}Te (Kitao, 2001) isotopes. The ground state bands up to 18 units of angular momentum are investigated for the moment of inertia and square of rotational energy in even $^{110\text{-}118}$Te nuclei. In Figure 1 the moments of inertia are plotted as a function of the even neutron number. It is shown that $2\vartheta/\hbar^2$ as a function of neutrons do not change up to spin 4^+. The $2\vartheta/\hbar^2$ value remains the same up to 8^+ levels for ^{110,112}Te isotopes and then gradually increases in $^{114\text{-}118}$Te isotopes. At 10^+ level the moment of inertia rapidly increases from N = 58 to 60 and then remain constant up to neutron 66. We state that at high spin states (12^+ to 18^+) the $2\vartheta/\hbar^2$ values decreases from N = 58 to 66 in even Te isotopes.

Table 2: Separation energy S_{2n}^{N+1} (MeV) of $^{112\text{-}118}$ Te isotopes

Isotopes	E_B(Z,N)MeV	E_B(Z,N-2) MeV	S_{2n}^{N+1} (MeV)
^{112}Te	928.890	900.550	28.340
^{114}Te	949.930	928.890	21.040
^{116}Te	970.010	949.930	20.080
^{118}Te	988.850	970.010	18.840

Table 3: Fermi energy λ(N,I) MeV of even $^{112\text{-}118}$Te isotopes

Spin(I)	^{112}Te	^{114}Te	^{116}Te	^{118}Te
2	-11.154	-10.511	-10.055	-9.457
4	-11.149	-10.526	-10.088	-9.46
6	-11.172	-10.564	-10.085	-9.435
8	-11.169	-10.617	-10.091	-9.429
10	-11.422	-10.537	-10.051	-9.432
12	-11.136	-10.523	-9.943	-9.373

The moments of inertia for even $^{110\text{-}118}$Te as a function of the square of rotational energy are plotted in Figure 2. In the region between 2^+ to 18^+ units of angular momentum, $2\vartheta/\hbar^2$ is observed in the yrast band of $^{110\text{-}118}$Te isotopes. In the lowest order according to the variable moment of inertia (VMI) model, this should give a straight line in the plot of inertia $2\vartheta/\hbar^2$ as a function of ω^2. The back-bending phenomena can be most easily demonstrated if one plots the moment of inertia as a function of ω^2. It is clear that the back-bending behaviour can be observed for each isotope except ^{118}Te. In ^{118}Te isotopes where a peculiar result was found, the square of rotational energy decreases as the moment of inertia increases. The $^{110\text{-}116}$Te isotopes show back-bending at I = 10^+, 8^+, 4^+, 4^+ levels, respectively. The moment of inertia increases rapidly at 12^+ states for even neutron N = 58, 60, 62, 64 of Te

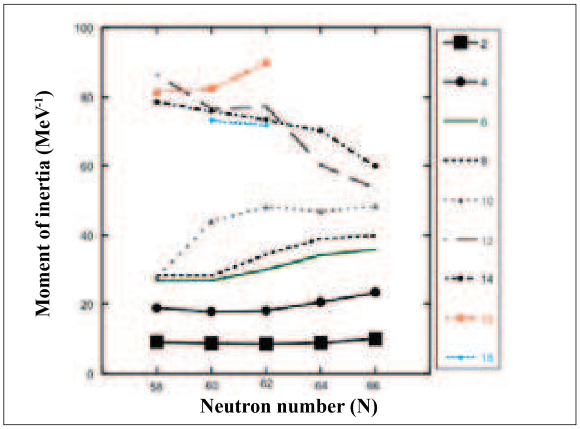

Figure 1: Moments of inertia (MeV^{-1}) versus even neutron number N = 58-66 of $^{110\text{-}118}$Te isotopes

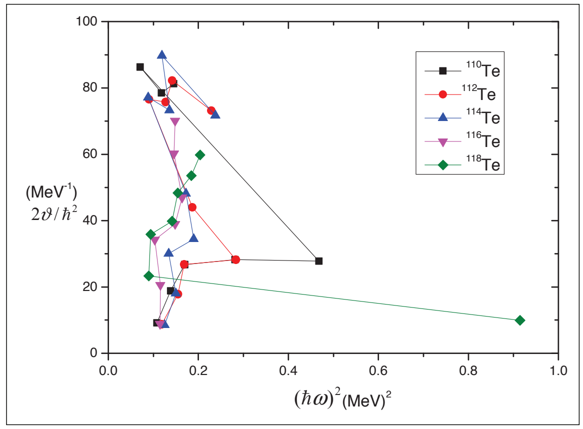

Figure 2: $2\vartheta/\hbar^2$ as a function of square of rotational energy $(\hbar\omega)^2$ of even $^{110\text{-}118}$Te isotopes

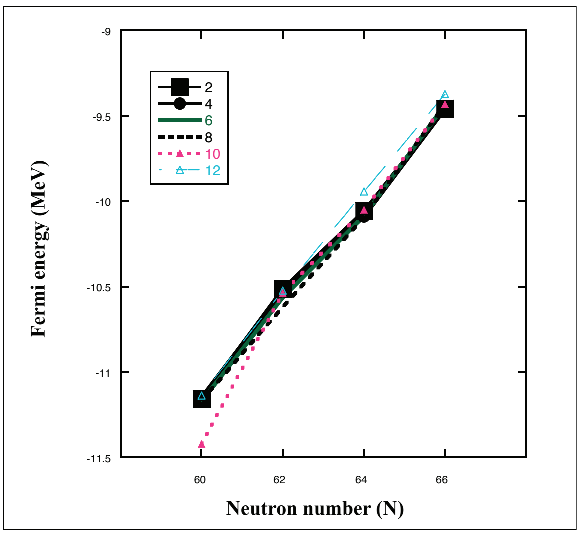

Figure 3: Fermi energies as a function of even neutron N = 60-66 of $^{112\text{-}118}$Te isotopes

isotopes. The results have been presented on collective ΔI = 2 ground band level sequence for the variation of shapes for Te isotopes with even neutron from N = 58−64. The back-bending phenomena appears clearly in the diagram $2\vartheta/\hbar^2$ vs. $(\hbar\omega)^2$ due to the crossing of two bands, which have different moments in a yrast line. This means that the interaction between two bands is small then one obtains a sudden transition which produce the beak-bending.

Fermi energies

The results have been confirmed in gauge space plot, which shows a back-bending behaviour when a change in deformation occurs. The separation energy was obtained using the equation (4) and has been presented in Table 2 (Kratsov, 1974). The Fermi energies λ(N,I) of even $^{112\text{-}118}$Te (Mansour & Saad, 2009) have been calculated from equation (3). The comparisons of Fermi energies of Te isotopes with even neutron N = 60 − 66 in gauge space for different states are presented in Figure 3. The Fermi energy of those nuclei at different levels (2^+, 4^+, 6^+, …. 12^+) are given in Table 3. The change of deformation can be explored through the so-called 'gauge-plots' of Fermi energy versus the even neutron number. Figure 3 shows the back-bending between the spherical nuclei with N< 62 and deformed nuclei with higher neutron number; the λ(N,I) curve reflects the level density at the Fermi surface.

CONCLUSION

In the present study we have focused on the analogy between the rotational energy in ordinary space and the Fermi energy in gauge space of even $^{110\text{-}118}$Te isotopes. The back-bending phenomena in ordinary space is compared with gauge space for the Fermi energies up to levels 12^+. The ^{118}Te isotopes were found to show an abnormal behaviour since the square of rotational energy decreases as the moment of inertia increases. The results of this study are extremely useful for compiling the nuclear data table.

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