J. Natn.Sci.Foundation Sri Lanka 2006 34(3): 143-148

RESEARCH ARTICLE

Relativistic aberration of Mixed number Lorentz Transformation

Md. Shah Alam and Md. Didar Chowdhury
[1] *Department of Physics, Shahjalal University of Science and Technology, Sylhet, Bangladesh.*
[2] *Department of Physics, Govt. Alia Madrasah, Sylhet, Bangladesh.*

Revised: 26 February 2006 ; Accepted: 23 May 2006

Abstract: Mixed number is the sum of a scalar and a vector. It has satisfactory algebra. In terms of Mixed number, we have derived the Lorentz Transformation, which we call the Mixed number Lorentz Transformation. The Mixed number Lorentz Transformation is better than the most general Lorentz Transformation. In this paper we have discussed the phenomenon of relativistic aberration by special, most general and Mixed number Lorentz Transformation. It has been observed that the explanation of relativistic aberration by Mixed number Lorentz Transformation is easier than the explanation of the same by most general Lorentz Transformation.

Key Words: Mixed number, Mixed number Lorentz Transformation, Relativistic-aberration.

1. INTRODUCTION

1.1 Mixed number

Mixed number[1,2] α is the sum of a scalar x and a vector **A**. i.e. $\alpha = x + \mathbf{A}$. It has satisfactory mathematical tools.[3] We have given below the summary of the mathematical tools of Mixed number algebra.

Two Mixed numbers α and β are equal if x = y and **A** = **B**. The addition of two Mixed numbers α and β can be written as:

$$\alpha + \beta = (x + y) + (\mathbf{A} + \mathbf{B}) \qquad \ldots\ldots(1)$$

where $\alpha = x + \mathbf{A}$ and $\beta = y + \mathbf{B}$

Mixed number $\alpha = x + \mathbf{A} = (x + A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k})$ where x, A_1, A_2 and A_3 are scalar values and **i**, **j** and **k** are basis or unit Mixed numbers. The properties of the basis Mixed numbers are $\mathbf{ii} = \mathbf{jj} = \mathbf{kk} = 1$ i.e. $\mathbf{i}^2 = 1$, $\mathbf{j}^2 = 1$, $\mathbf{k}^2 = 1$ and $\mathbf{ij} = i\,\mathbf{k}$, $\mathbf{jk} = i\,\mathbf{i}$, $\mathbf{ki} = i\,\mathbf{j}$, $\mathbf{ji} = -i\,\mathbf{k}$, $\mathbf{kj} = -i\,\mathbf{i}$, $\mathbf{ik} = -i\,\mathbf{j}$ where $i = \sqrt{(-1)}$.

They have the following multiplication table.

	1	i	j	k
1	1	i	j	k
i	i	1	ik	- ij
j	j	- ik	1	ii
k	k	ij	- ii	1

The product of two Mixed numbers α and β is defined as:

$$\alpha\beta = (x + \mathbf{A})(y + \mathbf{B}) =$$

$$xy + \mathbf{A.B} + x\mathbf{B} + y\mathbf{A} + i\mathbf{A}\times\mathbf{B} \qquad \ldots\ldots(2)$$

where **A.B** is the scalar product and **A×B** is the vector product of the vectors **A** and **B**. The product of Mixed numbers is associative.

$$\text{i.e. } (\alpha\beta)\gamma = \alpha(\beta\gamma) \qquad \ldots\ldots(3)$$

where $\gamma = z + \mathbf{C}$ is another Mixed number.
Taking x = y = 0 we get from equation (2):

$$\mathbf{A} \otimes \mathbf{B} = \mathbf{A.B} + i\mathbf{A} \times \mathbf{B} \qquad \ldots\ldots(4)$$

[The symbol ⊗ is chosen for Mixed product.]
This product is called Mixed product of vectors.[4]

1.2 Lorentz Transformation

1.2a Special Lorentz Transformation

Let us consider two inertial frames of references S and S′ where the frame S is at rest and the frame S′ is moving along X-axis

* Corresponding author

with velocity v with respect to S frame. The space and time coordinates of S and S′ are (x,y,z,t) and (x′,y′,z′,t′) respectively. We have assumed that the origins of both the frames S and S′ coincide at the time t = t′ = 0. The relation -ship between the coordinates of S and S′, which is called special Lorentz Transformation, can be written as[5]

$$\left.\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx) \end{aligned}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (5)$$

[where c = 1 and $\gamma = \dfrac{1}{\sqrt{(1 - v^2/c^2)}}$]

and

$$\left.\begin{aligned} x &= \gamma(x' + vt') \\ y &= y' \\ z &= z' \\ t &= \gamma(t' + vx') \end{aligned}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (6)$$

1.2b Most general Lorentz Transformation

When the velocity **v** of S′ with respect to the S is not along X-axis i.e. the velocity **v** has three components v_x, v_y and v_z then the relation between the coordinates of S and S′, which is called most general Lorentz Transformation, can be written as[6]

$$\left.\begin{aligned} \mathbf{x}' &= \mathbf{x} + \mathbf{v}[\{(\mathbf{x.v})/v^2\}(\gamma - 1) - t\gamma] \\ t' &= \gamma(t - \mathbf{x.v}) \end{aligned}\right| \quad \ldots\ldots\ldots\ldots \ (7)$$

where c = 1 and

$$\gamma = \frac{1}{\sqrt{(1 - v^2/c^2)}}$$

By using these Transformation equations addition of velocities can be written as

$$\mathbf{u}' = \frac{\mathbf{x}'}{t'} = \frac{\mathbf{x} + \mathbf{v}\,[\{(\mathbf{x.v})/v^2\}(\gamma - 1) - t\,\gamma]}{\gamma\,(t - \mathbf{x.v})} \quad \ldots\ldots\ldots \ (8)$$

Dividing numerator and denominator of the R.H.S. of equation (8) by t we get

$$\mathbf{u}' = \frac{\mathbf{x}'}{t'} = \frac{\mathbf{u} + \mathbf{v}[\{(\mathbf{u.v})/v^2\}(\gamma - 1) - \gamma]}{\gamma\,(1 - \mathbf{u.v})} = \mathbf{v} \oplus \mathbf{u} \quad \ldots\ldots \ (9$$

where **u** = **x**/t and the symbol ⊕ denotes the Lorent sum of velocities.

If v_x, v_y, v_z denote the components of the velocity of th system S′ relative to S, then equation (7) can be written a

$$x' = \{1 + (\gamma - 1)v_x^2/v^2\}x + (\gamma - 1)v_x v_y\, y/v^2 + (\gamma - 1)v_x v_z\, z/v^2 - v_x\gamma t$$

$$y' = (\gamma - 1)v_y v_x\, x/v^2 + \{1 + (\gamma - 1)v_y^2/v^2\}y + (\gamma - 1)v_y v_z\, z/v^2 - v_y\gamma t$$

$$z' = (\gamma - 1)v_z v_x\, x/v^2 + (\gamma - 1)v_z v_y\, y/v^2 + \{1 + (\gamma - 1)v_z^2/v^2\}z - v_z\gamma t$$

$$t' = -\gamma v_x x - \gamma v_y y - \gamma v_z z + \gamma t \quad \ldots\ldots\ldots \ (1$$

1.2c Mixed number Lorentz Transformation

Mixed number Lorentz Transformation can be generat using Mixed number algebra.[7]

From equation (5) we can write

$$t' + x' = \gamma\,(t - vx + x - vt)$$

$$t' + x' = \gamma\,\{(t + x) - v(t + x)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (1$$

Using (t′ + x′) = p′ and (t + x) = p in equation (11) v can write

$$p' = \gamma\,(p - pv) \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (1$$

From equation (6) we can write

$$t + x = \gamma\,(t' + vx' + x' + vt')$$

$$t + x = \gamma\,\{(t' + x') + v(t' + x')\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (1$$

Using (t′ + x′) = p′ and (t + x) = p in equation (13) we c write

$$p = \gamma\,(p' + p'v) \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (1$$

When the velocity **v** of S′ with respect to the S is not along axis i.e. the velocity **v** has three components v_x, v_y and v_z. I in this case **z** and **z′** be the space part in S and S′ fra respectively. In this case equation (12) can be written as

$$P' = \gamma\,(P - P\mathbf{v})$$

where P′ = (t′ + **z′**), and P = (t + **z**) are two Mixed numb

$$\therefore\ (t' + \mathbf{z}') = \gamma\ \{(t + \mathbf{z}) - (t + \mathbf{z})\mathbf{v}\}$$

$$\text{or, } (t' + \mathbf{z}') = \gamma\,\{(t + \mathbf{z}) - (t + \mathbf{z})\,(0 + \mathbf{v})\} \quad \ldots\ldots\ldots\ldots\ (1$$

Using equation (2) we can write

$$(t + \mathbf{z})(0 + \mathbf{v}) = \mathbf{z}.\mathbf{v} + t\mathbf{v} + i\mathbf{z}\times\mathbf{v} \quad \ldots\ldots\ldots\ldots(16)$$

From equation (2) and (15) we get

$$(t' + \mathbf{z}') = \gamma\{t + \mathbf{z} - (\mathbf{z}.\mathbf{v} + t\mathbf{v} + i\mathbf{z}\times\mathbf{v})\}$$

$$\text{or, } (t' + \mathbf{z}') = \gamma(t - \mathbf{z}.\mathbf{v}) + \gamma(\mathbf{z} - t\mathbf{v} - i\mathbf{z}\times\mathbf{v}) \quad \ldots\ldots (17)$$

Equating the scalar and vector parts of equation (17) we can write:

$$t' = \gamma(t - \mathbf{z}.\mathbf{v}) \text{ and} \quad \ldots\ldots\ldots\ldots (18)$$

$$\mathbf{z}' = \gamma(\mathbf{z} - t\mathbf{v} - i\,\mathbf{z}\times\mathbf{v})$$

Similarly it can be shown that

$$t = \gamma(t' + \mathbf{z}'.\mathbf{v}) \text{ and} \quad \ldots\ldots\ldots\ldots (19)$$

$$\mathbf{z} = \gamma(\mathbf{z}' + t'\mathbf{v} + i\mathbf{z}'\times\mathbf{v})$$

Equations (18) and (19) are the Mixed number Lorentz Transformation.

From equation (19) dividing **z** by t we get

$$\frac{\mathbf{z}}{t} = \frac{(\mathbf{z}' + t'\mathbf{v} + i\mathbf{z}'\times\mathbf{v})}{(t' + \mathbf{z}'.\mathbf{v})} \quad \ldots\ldots\ldots\ldots (20)$$

Dividing numerator and denominator of the R.H.S. of equation (20) by t′ we get

$$\frac{\mathbf{z}}{t} = \frac{\{(\mathbf{z}'/t') + \mathbf{v} + i(\mathbf{z}'/t')\times\mathbf{v}\}}{\{1 + (\mathbf{z}'/t').\mathbf{v}\}}$$

$$= \frac{\mathbf{u} + \mathbf{v} + i\,\mathbf{u}\times\mathbf{v}}{1 + \mathbf{u}.\mathbf{v}} \qquad (\text{where } \mathbf{u} = \mathbf{z}'/t')$$

$$\text{or, } \quad \mathbf{w} = \mathbf{v} \oplus \mathbf{u} = \frac{\mathbf{u} + \mathbf{v} + i\,\mathbf{u}\times\mathbf{v}}{1 + \mathbf{u}.\mathbf{v}} \quad \ldots\ldots\ldots\ldots (21)$$

where $\mathbf{w} = \mathbf{z}/t$

Equation (21) is the relativistic velocity addition theorem of Mixed number Lorentz Transformation. The Mixed number Lorentz Transformation is better than the most general Lorentz Transformation.[7]

2. ABERRATION

The phenomenon of aberration results "The speed of light is independent of the medium of transmission; but the direction of light rays depends on the motion of the source emitting light relative to the observer."

2.1 Proof of aberration by special Lorentz Transformation

The earth revolves round the sun in its orbit, therefore we may imagine the sun to be system S while the earth in system S′ is moving with velocity **v** relative to system S along positive direction of common X – axis. Let the light from a star P be observed by observers O and O′ in system S and S′ respectively where the frame S′ is moving with velocity **v** with respect to S frame along X-axis. Let the angles made by a light ray in X-Y plane from the star P at any instant in two systems at O and O′ be φ (φ= <POX) and φ' (φ'= <PO′X′) respectively.

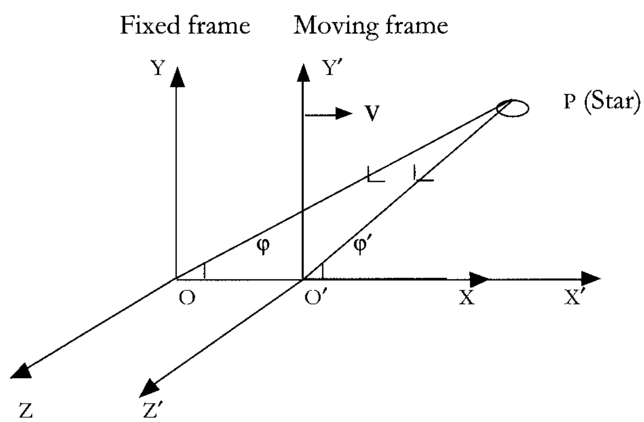

Figure 1: Direction of light rays observed from a fixed frame S and a moving frame S′(The frame S′ is moving relative to S along common X-axis).

It can be shown that[8]:

$$\tan\varphi' = \frac{\tan\varphi\,\sqrt{(1-\beta^2)}}{1-\beta\sec\varphi} \quad \ldots\ldots\ldots\ldots (22)$$

(where $\beta = v/c$)

and

$$\tan\varphi = \frac{\tan\varphi'\,\sqrt{(1-\beta^2)}}{1-\beta\sec\varphi'} \quad \ldots\ldots\ldots\ldots (23)$$

From equations (22) and (23) it is clear that φ and φ' are not the same in the two systems; i.e. the direction of light rays depends upon the relative motion of source and observer; therefore the explanation of the phenomenon of aberration is obvious. Equation (23) gives the exact relativistic aberration formula for special Lorentz Transformation.

2.2 Proof of aberration by most general Lorentz Transformation

Let us now consider the light from a star P observed by observers O and O′ in system S and S′ respectively. The frame S′ is moving with velocity **v** relative to S along arbitrary direction as shown in Figure 2. Let the angles made by a light ray in X-Y plane from the star P at any instant in two systems at O and O′ be φ (φ =<POX) and φ' (φ' = <PO′X′) respectively. According to the description the angle φ is same in Figure 1 and Figure 2 but the angle φ' is different.

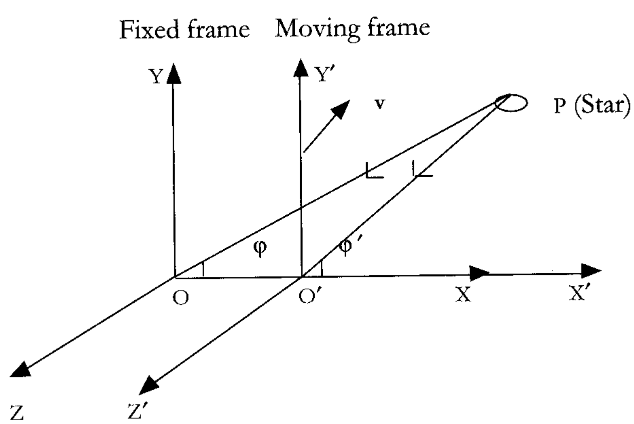

Figure 2: Direction of light rays observed from a fixed frame S and a moving frame S′(The frame S′ is moving relative to S along arbitrary direction).

Differentiating both sides of equation (10) we get

$$dx' = \{1 + (\gamma - 1)v_x^2/v^2\}dx + (\gamma - 1)\, v_x v_y/v^2\, dy + (\gamma - 1)v_x v_z/v^2\, dz - v_x\gamma dt \qquad \text{......... (24)}$$

$$dy' = (\gamma - 1)v_y v_x/v^2 dx + \{1 + (\gamma - 1)v_y^2/v^2\}dy + (\gamma - 1)v_y v_z/v^2 dz - v_y\gamma dt \qquad \text{......... (25)}$$

$$dt' = -\gamma v_x dx - \gamma v_y dy - \gamma v_z dz + \gamma dt \qquad \text{.................. (26)}$$

From equation (24) and (26) on dividing, we get

$$u'_x = \frac{dx'}{dt'} = \frac{\{1 + (\gamma - 1)v_x^2/v^2\}dx + (\gamma - 1)v_x v_y/v^2\, dy + (\gamma - 1)v_x v_z/v^2\, dz - v_x\gamma dt}{-\gamma v_x dx - \gamma v_y dy - \gamma v_z dz + \gamma dt}$$

$$\text{or, } u'_x = \frac{\{1 + (\gamma - 1)v_x^2/v^2\}\, u_x + (\gamma - 1)v_x v_y/v^2\, u_y + (\gamma - 1)v_x v_z/v^2\, u_z - v_x\gamma}{-\gamma v_x u_x - \gamma v_y u_y - \gamma v_z u_z + \gamma} \qquad \text{........(27)}$$

Again from equation (25) and (26) on dividing, we get

$$\text{or, } u'_y = \frac{dy'}{dt'} = \frac{(\gamma - 1)v_y v_x/v^2 dx + \{1 + (\gamma - 1)v_y^2/v^2\}dy + (\gamma - 1)v_y v_z/v^2 dz - v_y\gamma dt}{-\gamma v_x dx - \gamma v_y dy - \gamma v_z dz + \gamma dt}$$

$$\text{or, } u'_y = \frac{(\gamma - 1)v_y v_x/v^2 u_x + \{1 + (\gamma - 1)v_y^2/v^2\}\, u_y + (\gamma - 1)v_y v_z/v^2\, u_z - v_y\gamma}{-\gamma v_x u_x - \gamma v_y u_y - \gamma v_z u_z + \gamma} \qquad \text{......... (28)}$$

From equation (27) and (28) we can write

$$\frac{u'_y}{u'_x} = \frac{(\gamma - 1)v_y v_x/v^2 u_x + \{1 + (\gamma - 1)v_y^2/v^2\}\, u_y + (\gamma - 1)v_y v_z/v^2\, u_z - v_y\gamma}{\{1 + (\gamma - 1)v_x^2/v^2\}\, u_x + (\gamma - 1)v_x v_y/v^2\, u_y + (\gamma - 1)v_x v_z/v^2\, u_z - v_x\gamma} \qquad \text{......... (29)}$$

If we consider two-dimensional case, then we can write from equation (29)

$$\frac{u'_y}{u'_x} = \frac{(\gamma - 1)v_y v_x/v^2 u_x + \{1 + (\gamma - 1)v_y^2/v^2\}\, u_y - v_y\gamma}{\{1 + (\gamma - 1)v_x^2/v^2\}\, u_x + (\gamma - 1)v_x v_y/v^2\, u_y - v_x\gamma} \qquad \text{...... (30)}$$

In this case the starlight travelling in x-y plane with velocity c, has components, c cos(π +φ) and c cos(π +φ') along positive direction of X-axis in system S and S′ respectively. Also those c sin(π +φ) and c sin(π+φ') along positive direction of Y-axis in system S and S′ respectively. Thus we have

$$\begin{aligned} u_x &= c\cos(\pi + \varphi) = -\,c\cos\varphi \\ u_y &= c\sin(\pi + \varphi) = -\,c\sin\varphi \\ u'_x &= c\cos(\pi + \varphi') = -\,c\cos\varphi' \\ u'_y &= c\sin(\pi + \varphi') = -\,c\sin\varphi' \end{aligned} \qquad \text{.......................(31)}$$

Substituting the values from equation (31) into equation (30)

$$\frac{-\,c\sin\varphi'}{-\,c\cos\varphi'} = \frac{(\gamma - 1)v_y v_x/v^2(-\,c\cos\varphi) + \{1 + (\gamma - 1)v_y^2/v^2\}(-\,c\sin\varphi) - v_y\gamma}{\{1 + (\gamma - 1)v_x^2/v^2\}(-\,c\cos\varphi) + (\gamma - 1)v_x v_y/v^2\,(-\,c\sin\varphi) - v_x\gamma}$$

$$\text{or, } \tan\varphi' = \frac{(\gamma - 1)v_y v_x/v^2 + \{1 + (\gamma - 1)v_y^2/v^2\}(\tan\varphi) + (v_y/c)\gamma\sec\varphi}{\{1 + (\gamma - 1)v_x^2/v^2\} + (\gamma - 1)v_x v_y/v^2 \tan\varphi + (v_x/c)\gamma\sec\varphi} \qquad \text{............... (32)}$$

Using c = 1 we get from equation (32),

$$\tan\varphi' = \frac{(\gamma - 1)v_y v_x/v^2 + \{1 + (\gamma - 1)v_y^2/v^2\}(\tan\varphi) + (v_y)\gamma \sec\varphi}{\{1 + (\gamma - 1)v_x^2/v^2\} + (\gamma - 1)v_x v_y/v^2 \tan\varphi + (v_x)\gamma \sec\varphi} \quad .... (33)$$

From equation (33) it is clear that φ and φ' are not the same in the two systems, i.e., the direction of light rays depends upon the relative motion of the source and observer. Therefore, we can explain the phenomenon of relativistic aberration by most general Lorentz Transformation. Equation (33) gives the relativistic aberration formula for most general Lorentz Transformation.

2.3 Proof of aberration by Mixed number Lorentz Transformation

Let us consider the same phenomenon as described in Figure 2. Considering $z_x = x$, $z_y = y$, $z_z = z$, $z'_x = x'$, $z'_y = y'$ and $z'_z = z$ equation (18) can be written as

$$\left.\begin{aligned} x' &= \gamma\{x - tv_x - i(yv_z - zv_y)\} \\ y' &= \gamma\{y - tv_y - i(zv_x - xv_z)\} \\ z' &= \gamma\{z - tv_z - i(xv_y - yv_x)\} \\ t' &= \gamma(t - xv_x - yv_y - zv_z) \end{aligned}\right| \quad .... (34)$$

Differentiating both sides of equation (34) we get

$$dx' = \gamma\{dx - v_x dt - i(v_z dy - v_y dz)\} \quad .... (35)$$

$$dy' = \gamma\{dy - v_y dt - i(v_x dz - v_z dx)\} \quad .... (36)$$

$$dt' = \gamma(dt - v_x dx - v_y dy - v_z dz) \quad .... (37)$$

From equation (35), (36) and (37) we can write

$$\frac{dx'}{dt'} = \frac{\gamma\{dx - v_x dt - i(v_z dy - v_y dz)\}}{\gamma(dt - v_x dx - v_y dy - v_z dz)}$$

$$\text{or. } u'_x = \frac{dx'}{dt'} = \frac{\gamma\{dx/dt - v_x - i(v_z dy/dt - v_y dz/dt)\}}{\gamma(1 - v_x dx/dt - v_y dy/dt - v_z dz/dt)}$$

$$\text{or. } u'_x = \frac{\gamma\{u_x - v_x - i(v_z u_y - v_y u_z)\}}{\gamma(1 - v_x u_x - v_y u_y - v_z u_z)} \quad .... (38)$$

$$\text{and } \frac{dy'}{dt'} = \frac{\gamma\{dy - v_y dt - i(v_x dz - v_z dx)\}}{\gamma(dt - v_x dx - v_y dy - v_z dz)}$$

$$\text{or. } u'_y = \frac{dy'}{dt'} = \frac{\gamma\{dy/dt - v_y - i(v_x dz/dt - v_z dx/dt)\}}{\gamma(1 - v_x dx/dt - v_y dy/dt - v_z dz/dt)}$$

$$\text{or. } u'_y = \frac{\{u_y - v_y - i(v_x u_z - v_z u_x)\}}{(1 - v_x u_x - v_y u_y - v_z u_z)} \quad .... (39)$$

Dividing Equation (39) by equation (38) we get

$$\frac{u'_y}{u'_x} = \frac{\{u_y - v_y - i(v_x u_z - v_z u_x)\}}{\{u_x - v_x - i(v_z u_y - v_y u_z)\}} \quad .... (40)$$

If we consider the two-dimensional case, then we can write from equation (40)

$$\frac{u'_y}{u'_x} = \frac{(u_y - v_y)}{(u_x - v_x)} \quad .... (41)$$

From equation (31) and (41) we can write

$$\frac{-c\sin\varphi'}{-c\cos\varphi'} = \frac{(-c\sin\varphi - v_y)}{(-c\cos\varphi - v_x)}$$

$$\text{or, } \tan\varphi' = \frac{c\sin\varphi + v_y}{c\cos\varphi + v_x}$$

Considering c = 1 we get

$$\text{or, } \tan\varphi' = \frac{\sin\varphi + v_y}{\cos\varphi + v_x} \quad .... (42)$$

$$\text{or, } \tan\varphi' = \frac{\tan\varphi + v_y \sec\varphi}{1 + v_x \sec\varphi} \quad .... (43)$$

Table 3: Comparison of aberration in special, most general and Mixed number Lorentz Transformation

	Special Lorentz Transformation	Most general Lorentz Transformation	Mixed number Lorentz Transformation
Relativistic aberration formula	$\tan\varphi' = \dfrac{\tan\varphi\sqrt{(1-\beta^2)}}{1-\beta\sec\varphi}$	$\tan\varphi' = \dfrac{(\gamma-1)v_y v_x/v^2+\{1+(\gamma-1)v_y^2/v^2\}\tan\varphi + (v_y)\gamma\sec\varphi}{\{1+(\gamma-1)v_x^2/v^2\}+(\gamma-1)v_x v_y/v^2\tan\varphi + (v_x)\gamma\sec\varphi}$	$\tan\varphi' = \dfrac{\tan\varphi =+ v_y\sec\varphi}{1 + v_x\sec\varphi}$

From equation (43) it is clear that for any value of **v** (except **v** = 0) φ and φ′ are not the same in the two systems; i.e. the direction of light rays depends upon the relative motion of source and observer; therefore the explanation of the phenomenon of aberration is obvious. Equation (43) gives the exact relativistic aberration formula for Mixed number Lorentz Transformation.

CONCLUSION

The phenomenon of relativistic aberration is clearly explained by special, most general and Mixed number Lorentz Transformation. We have observed that the relativistic aberration formula for Mixed number Lorentz Transformation is simpler than relativistic aberration formula for most general Lorentz Transformation. Therefore we can easily calculate the relativistic aberration for the case where the inertial frame is moving in any arbitrary direction instead of X-axis by the help of relativistic aberration formula for Mixed number Lorentz Transformation.

Acknowledgement

We are grateful to Mushfiq Ahmad, Dept. Physics, University of Rajshahi, Rajshahi, Bangladesh and Prof. Habibul Ahsan, Dept. of Physics, Shahjalal University of Science and Technology, Sylhet, Bangladesh for their help and advice.

References

1. Md. Shah Alam (2003). "Study of Mixed Number." *Proceedings of the Pakistan Academy of sciences* **37**(1): 19-122.
2. Md. Shah Alam (2001). Comparative study of Quaternions and Mixed number. *Journal of Theoretics* **3**(6).
3. Md. Shah Alam, Ahsan M. H. & Ahmad Mushfiq (2005). "Mathematical Tools of Mixed Number algebra." *Journal of the National Science Foundation of Sri Lanka* **33**(1):43-49.
4. Md. Shah Alam (2001). "Mixed Product of Vectors." *Journal of Theoretics* **3**(4).
5. Resnick Robert (1994). *Introduction to special relativity.* Wiley Eastern Limited, New Deihi.
6. Moller C. (1992).The *Theory of Realativity.* Oxford University press.
7. Md. Alam Shah & Ahsan M. H. (2003). Mixed number Lorentz Transformation. *Physics Essays* **16**(4): 530-536.
8. Prakash Satya (1993). *Relativistic Mechanics.* Pragati Prakashan.