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RESEARCH ARTICLE

A simple method for determining optimal fishing strategies using the Beverton and Holt yield per recruit model in fish stocks with parameter uncertainties

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Abstract: Parameter uncertainties of existing fish stock assessment models may lead to incorrect management recommendations on optimal fishing strategies especially in tropical fisheries. Therefore, it is advisable to consider the effect of uncertainties in the population parameters when formulating management recommendations. This paper attempts to evaluate the parameter uncertainties of the Beverton and Holt yield per recruit model to estimate optimal fishing strategies. A 10% variation on deterministic estimates of population parameters of *Amblygaster sirm* off the Western coastal waters of Sri Lanka was used to demonstrate the proposed method. Different magnitudes of perturbations of the estimated values of natural mortality, asymptotic weight and growth coefficient were considered and optimal management parameters of the Beverton and Holt yield per recruit model were determined. Multiple regression models were also formulated to describe the relationships between the uncertain input parameters and the optimal management parameters.

Key words: Beverton and Holt yield per recruit model, management recommendations, multiple regression, parameter uncertainties.

INTRODUCTION

A number of models have been developed to assess stock for prediction of optimal fishing strategies,[1-3] but there have been uncertainties associated with the outcomes from most models. This could either be associated with the process of estimation of population parameters or the basic assumptions made during the model development. This paper focuses on the former problem of the uncertainty in parameter estimation. It is understood that the uncertainties in estimation of population parameters might result in imprecise estimates of optimal fishing strategies and hence, erroneous management recommendations. [4] This study attempts to evaluate the impact of parameter uncertainties and the sensitivity of input parameters on the output of Beverton and Holt yield per recruit model [3] for determining optimal fishing strategies.

The Beverton and Holt yield per recruit (Y/R) model is one of the popular analytical models available for fish stock assessments. [5] This model can be described in the following form.

$$Y/R = F \exp(-M(t_c - t_r))\, W_\infty \sum_{n=0}^{3} \{[u_n / (F+M+nK)]\, [\exp(-(nK(t_c - t_o)))]\, [1-\exp(-(F+M+nK)(t_l - t_c))]\}$$

where,

$u_0 = 1; u_1 = -3; u_2 = 3; u_3 = -1$
F = instantaneous rate of fishing mortality
M = instantaneous rate of natural morality
t_c = age at first capture
t_r = age of recruitment
W_∞ = asymptotic weight
K = Von Bertalanffy growth coefficient
t_o = theoretical age at zero length
t_l = age of the oldest fish in stock.

Of the eight parameters of the above model, the only controllable variables are F and t_c (F is proportional to fishing effort while t_c is a function of gear selectivity). In general, M, W_∞ and K are relatively uncertain parameters compared to the age parameters (i.e. t_o, t_l, t_r), hence the former are considered as the key parameters influencing management recommendations. Therefore, quantifying these uncertainties in the process of assessment of stocks is essential to ensure the validity of management recommendations. This study presents a mechanism to determine the optimal fishing strategy in the presence of parameter uncertainties using the Beverton and Holt yield per recruit model. The estimated population parameters of *Amblygaster sirm* (Clupeidae) off Western coastal waters of Sri Lanka by Karunasinghe[6]

Table 1: The parameter estimates made for *A. sirm* from the western coastal waters of Sri Lanka. [6]

Parameter	Value
Asymptotic length (L_∞)	25.4 cm
Growth constant (K)	0.98 year $^{-1}$
Instantaneous rate of fishing mortality (F)	3.36 year $^{-1}$
Instantaneous rate of natural mortality(M)	0.96 year $^{-1}$
Length at first capture (L_c)	14.95 cm
Length at recruitment (L_r)	10.5 cm
The maximum length of the fish in the stock (L_l)	23.5 cm

(Table 1) were used to demonstrate the proposed method. *A. sirm* is a small pelagic fish, found in the coastal waters of the Western Indian Ocean.[7] It is highly abundant off the southern, western and the northwestern coastal waters of Sri Lanka and is frequently found in the catches of gillnets, purse seines and occasionally in beach seines operated at depths varying from 5-70 m .[8] The population parameters of *A. sirm* off the western coastal waters of Sri Lanka estimated by various authors showed substantial differences [6,9,10]

This paper also attempts to develop appropriate relationships between the uncertain parameters (M, W_∞ and K) and optimal management parameters (maximum yield per recruit and optimum age at first capture).

METHODS AND MATERIALS

Theoretical age at length zero (t_o) was estimated using the empirical equation described by Pauly.[11]

$$\text{Log}(-t_o) = -0.392 - 0.275 \text{ Log } L_\infty - 1.038 \text{ K}$$

where L_∞ is asymptotic total length (cm); K is on annual basis.

The following length-weight relationship[6] was used to estimate W_∞ from L_∞.

$$W = 0.01L^3$$

The length at first capture (L_c), length at recruitment (L_r) and the maximum length of the fish in the stock (L_l) were converted into corresponding ages (t_c, t_r and t_l) using the inverse von Bevtalanffy growth equation[12],

$$t = t_o - 1/K \ln(1 - L_t / L_\infty)$$

where L_t = length at age t.

Table 2: Other estimated parameters of the Beverton and Holt yield per recruit model

Parameter	Value
Asymptotic weight (W_∞)	163.87 g
Age at zero length (t_o)	-0.016 years
Age at first capture (t_c)	0.89 years
Age of recruitment (t_r)	0.53 years
Age of the oldest fish in stock (t_l)	2.63 years

The above estimates and the estimated values for other population parameters from the previous study (Table 1) were used as the initial inputs to the Beverton and Holt yield per recruit model; ±10% variations from existing values of M, W_∞ and K were assumed in the study. Thereby, 21 values were obtained for each of these parameters by making 1% variation at a time over the considered range. It should be noted that there are 9261 (21x21x21) permutations for M, W_∞ and K. For each combination of M, W_∞ and K, the maximum yield per recruit (Ypr max), age at first capture corresponding to the Ypr max (opt t_c) and the fishing mortality corresponding to the Ypr max (F_{max}) were estimated by optimizing the Beverton and Holt yield per recruit function. In the optimization process, F and t_c were allowed to vary over the acceptable ranges from 0.1 to 5 and 0.1 to 4 respectively. In terms of both parameters the step size was set at 0.01. The other parameters of the Beverton and Holt yield per recruit model were fixed.

For estimation of optimal management parameter values over the above ranges of F and t_c for each combination of M, W_∞ and K, a computer program in Pascal Language was used (available from author upon request). For each value of M, two types of contour plots were drawn using the Matlab computer software. In the first contour plot, maximum yield per recruit was plotted against W_∞ and K. In the second contour plot, opt t_c was plotted against W_∞ and K.

A multiple regression model was fitted to describe the relationship between the Ypr max, K, M and W_∞ In this exercise, Ypr max was considered as the response variable while M, K and W_∞ were considered as the predictor variables.

$$\text{Ypr max} = ß_0 + ß_1 M + ß_2 W_\infty + ß_3 K + ß_4 KW_\infty + ß_5 KM$$

$$ß_6 MW_\infty + ß_7 KMW_\infty$$

where $ß_{0-7}$ are the parameters to be determined.

A multiple regression model was also fitted to predict the opt t_c. This multiple regression analysis was carried out using Microsoft Excel. In the process of

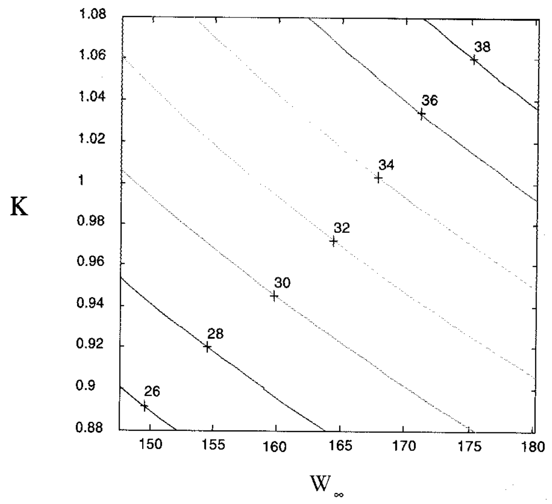

Figure 1: Maximum yield per recruit (Ypr max) contour as a function of asymptotic weight (W_∞) and growth constant (K) at the Natural mortality (M) of 0.86 for *A. sirm* in the west coast of Sri Lanka.

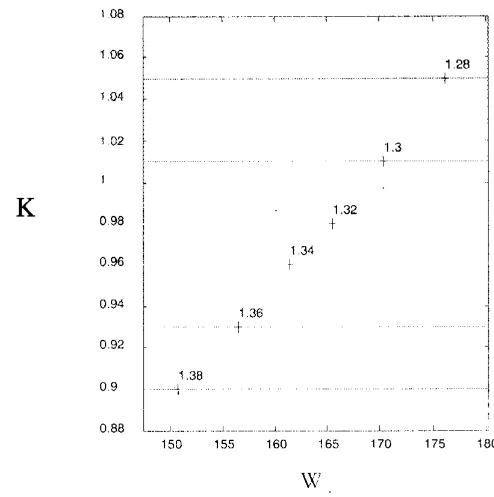

Figure 2: Optimal Age at the first capture (opt t_c) contours as a function of asymptotic weight (W_∞) and growth constant (K) at the natural mortality (M) of 0.86 for *A. sirm* in the west coast of Sri Lanka.

deriving the regression models, the total number of parameter combinations (9261) was taken into consideration.

RESULTS

The estimated values for W_∞, t_0, t_c, t_r and t_l for *A. sirm* based on the values in Karunasinghe[6] (Table 1) are given in Table 2. The estimated optimal yield per recruit based on the above study was 29.43 g. The corresponding opt t_c and F_{max} were 1.24 years and 5.0 year^{-1} respectively.

The contour plots of Ypr max and opt t_c against W_∞ and K for M = 0.86 only are shown in Figure 1 and Figure 2 respectively. All other contour plots for different values of M were similar in shape, but there were considerable differences between the two types of contour plots. Since the opt t_c contours are parallel to the W_∞ - axis, opt t_c is independent of W_∞.

The estimated minimum value for the Ypr max was 20.9 g. The corresponding combination of percentage perturbation of M, W_∞ and K was 10, -10 and -10% respectively. Similarly, the estimated maximum value for Ypr max was 40 g and the corresponding combination of percentage perturbation of M, W_∞ and K was – 10, 10 and 10%, respectively. Therefore, if the estimated values of M, W_∞ and K varied by ± 10% the Ypr max would vary from – 29 to 36% with respect to the initial prediction of Ypr max of the Beverton and Holt model.

The F_{max} was almost the same (5 year^{-1}) for all combinations. This implies that if the fishing mortality varies from 0.1 – 5 year^{-1}, the Ypr max is achieved only when the fishing mortality is equal to 5 year^{-1}. It should also be noted that for different combinations of M, W_∞ and K the estimated values for opt t_c were also different. As opt t_c is independent of W_∞ the minimum value for opt t_c could be obtained at the 10 % variation of M and K which is 1.12 years. Similarly, the maximum value for opt t_c could be obtained at -10 % perturbation of M and K (1.39 years). Therefore, the estimated range for the opt t_c was 1.12 – 1.39 years.

The analysis of variance indicated that the estimated coefficients of the terms KW_∞, MW_∞ and KMW_∞ (β_4, β_6 and β_7 respectively) were significant at the 99 % probability level (Table 3). The best model to derive Ypr max (R^2 = 0.99, Table 4) was:

$$\text{Ypr max} = 0.343\, KW_\infty - 0.055\, MW_\infty - 0.104\, KMW_\infty$$

It was noted that opt t_c depends upon K and M only. Therefore, a multiple regression model was fitted between opt t_c, M and K:

$$\text{opt } t_c = \alpha_0 + \alpha_1 M + \alpha_2 K + \alpha_3 MK$$

where α_{0-3} are the parameters to be determined. The statistical analysis indicated that all the coefficients were significant at the 99 % probability level (Table 5).

The best model to derive opt t_c (R^2 = 0.99: Table 6) was:

$$\text{opt } t_c = 3.048 - 1.2266M - 1.108\, K + 0.4896\, MK$$

Table 3: Parameter estimates of the regression model of maximum yield per recruit (Ypr max)

Parameter	Value	Standard Error	P – value
β_0 (Constant)	0.092	3.1807	0.98
β_1 (M)	-0.115	3.3067	0.97
β_2 (W_∞)	-0.007	0.0193	0.72
β_3 (K)	-0.095	3.2395	0.98
β_4 (K W_∞)	0.343	0.0197	5.45 E –67
β_5 (KM)	0.120	3.3677	0.97
β_6 (MW_∞)	-0.055	0.0201	0.006
β_7 (KMW_∞)	-0.104	0.0205	4.58 E -07

Table 4: Analysis of variance of the regression model of maximum yield per recruit (Ypr max)

	df	SS	MS	F	Significance F
Regression	7	106773.2	15253.31	2956574	0
Residual	9253	47.73732	0.005159		
Total	9260	106820.9			

Table 5: Parameter estimates of the regression model of optimal age at first capture (opt t_c)

Parameter	Value	Standard Error	P – value
α_0 (Constant)	3.048	0.0091	0.005
α_1 (M)	-1.2266	0.0094	4.5 E -25
α_2 (K)	-1.108	0.0092	0.001
α_3 (MK)	0.4896	0.0097	0.003

Table 6: Analysis of variance of the regression model of optimal age at first capture (opt t_c)

	df	SS	MS	F	Significance F
Regression	3	32.77033	10.92344	940090.5	0
Residual	9257	0.107562	1.16E-05		
Total	9260	32.8779			

DISCUSSION

Quantifying the uncertainties in fisheries stock assessment is essential to ensure the validity of management recommendations for sustainable use of fisheries resources. The present exercise attempted to describe the influence of parameter uncertainties on fishery resources evaluation to bring about optimal fishing strategies, using an example of available information on *A. sirm* fishery off the western coastal waters of Sri Lanka.

There is a substantial difference between the estimated population parameters (L_∞, K and M) of *A. sirm* from the western coastal waters of Sri Lanka between 1980-1990. The estimated values for L_∞ and K varied from 22.9 –26.0 cm and 0.95 – 2.38 year^{-1} respectively. [6,9,10] In addition, the estimated values for M varied from 0.96 - 2.12 year^{-1}. The uncertainty on optimal fishing strategy greatly depends on L_∞, K and M, while the other parameters such as t_l have little influence on the output. It should also be noted that the two models derived for optimal fishing strategies (i.e. two regression models derived to predict Ypr max and opt t_c) were based on input values of all parameters of the Beverton and Holt yield per recruit model. However, the best models of optimal fishing strategies (see results) consist of at most three parameters of W_∞, M and K. Therefore, estimation of Ypr max and opt t_c using these multiple regression models is much easier compared to the use of the full Beverton and Holt yield per recruit model as the information requirement is relatively low. Hence the initial cost of collecting fisheries data would also be lower.

The present work was conducted to evaluate the effects of parameter uncertainties and the sensitivity of the input parameters of the Beverton and Holt yield per recruit model on determination of the optimal fishing strategy for the *A. sirm* fishery in the western coastal waters of Sri Lanka. Therefore, these models are not directly applicable to any other species. However, there is a possibility of adopting a similar procedure for estimation of appropriate models for different species.

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