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MATHEMATICAL TOOLS OF MIXED NUMBER ALGEBRA

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Abstract: Mixed number is the sum of a scalar and a vector. The quaternion can also be written as the sum of a scalar and a vector but the product of mixed numbers and the product of quaternions are different. Mixed product is better than the quaternion product because Mixed product is directly consistent with Pauli matrix algebra and Dirac equation but the quaternion product is not directly consistent with Pauli matrix algebra and Dirac equation. In this paper we have represented the complete mathematical tools of mixed number algebra, and shown its potential value with actiral examples.

Key Words: mixed number, mixed product, quaternion product

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INTRODUCTION

Definition 1

Mixed number[1,2] α is the sum of a scalar x and a vector **A** like quaternions[3-5]
i.e. $\alpha = x + \mathbf{A}$

Lemma 1

Two Mixed numbers α and β are equal if
$x = y$ and $\mathbf{A} = \mathbf{B}$

Definition 2

The addition of two Mixed numbers α and β is

$$\alpha+\beta = (x+y) + (\mathbf{A} + \mathbf{B}) \qquad \text{......................(1)}$$

where $\alpha = x + \mathbf{A}$ and $\beta = y + \mathbf{B}$

Definition 3

Mixed number $\alpha = x + \mathbf{A} = (x + A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k})$ where x, A_1, A_2 and A_3 are scalar values and **i**, **j** and **k** are unit Mixed numbers

Lemma 1: The properties of the unit Mixed numbers are

$\mathbf{ii} = \mathbf{jj} = \mathbf{kk} = 1$ i.e. $\mathbf{i}^2 = 1, \mathbf{j}^2 = 1, \mathbf{k}^2 = 1$ and

$\mathbf{ij} = i\,\mathbf{k}$, $\mathbf{jk} = i\,\mathbf{i}$, $\mathbf{ki} = i\,\mathbf{j}$, $\mathbf{ji} = -i\,\mathbf{k}$, $\mathbf{kj} = -i\,\mathbf{i}$, $\mathbf{ik} = -i\,\mathbf{j}$ where $i = \sqrt{(-1)}$.

Lemma 2: They have the following multiplication table

	1	i	j	k
1	1	i	j	k
i	i	1	ik	- ij
j	j	- ik	1	ii
k	k	ij	- ii	1

Definition 4

The product of two Mixed numbers α and β is defined as

$$\alpha\beta = (x + \mathbf{A})(y + \mathbf{B}) = xy + \mathbf{A.B} + x\mathbf{B} + y\mathbf{A} + i\mathbf{A}\times\mathbf{B} \qquad \text{..............................(2)}$$

Lemma 1

The product of Mixed numbers is associative

$$\text{i.e.} (\alpha\beta)\gamma = \alpha(\beta\gamma) \qquad \text{..............................(3)}$$

where $\gamma = z + \mathbf{C}$ is another Mixed number

Proof:

$$\begin{aligned}(\alpha\beta)\gamma &= \{(x + \mathbf{A})(y + \mathbf{B})\}(z + \mathbf{C}) \\ &= (xy + \mathbf{A.B} + x\mathbf{B} + y\mathbf{A} + i\mathbf{A}\times\mathbf{B})(z + \mathbf{C}) \\ &= (xy + \mathbf{A.B})z + (x\mathbf{B} + y\mathbf{A} + i\mathbf{A}\times\mathbf{B}) \mathbf{.\,C} \\ &+ (xy + \mathbf{A.B})\mathbf{C} + z(x\mathbf{B} + y\mathbf{A} + i\mathbf{A}\times\mathbf{B}) + \\ &i\,(x\mathbf{B} + y\mathbf{A} + i\mathbf{A}\times\mathbf{B}) \times \mathbf{C}\end{aligned}$$

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$= xyz + (\mathbf{A.B})z + x\mathbf{B.C} + y\mathbf{A.C} + i(\mathbf{A}\times\mathbf{B}).\mathbf{C}$

$+ xy\mathbf{C} + (\mathbf{A.B})\mathbf{C} + z x\mathbf{B} + yz\mathbf{A} + i\, z(\mathbf{A}\times\mathbf{B})$

$+ ix\,(\mathbf{B}\times\mathbf{C}) + i\, y(\mathbf{A}\times\mathbf{C}) - (\mathbf{A}\times\mathbf{B})\times\mathbf{C}$

$= xyz + (\mathbf{A.B})z + x\mathbf{B.C} + y\mathbf{A.C} + i(\mathbf{A}\times\mathbf{B}).\mathbf{C}$

$+ xy\mathbf{C} + (\mathbf{A.B})\mathbf{C} + zx\mathbf{B} + yz\mathbf{A}$

$+ i\, z(\mathbf{A}\times\mathbf{B}) + ix\,(\mathbf{B}\times\mathbf{C}) + i\, y(\mathbf{A}\times\mathbf{C}) - \mathbf{B}(\mathbf{A.C})$

$+ \mathbf{A}(\mathbf{B.C})$

$= x\{yz + \mathbf{B.C} + y\mathbf{C} + z\mathbf{B} + i\,(\mathbf{B}\times\mathbf{C})\}$

$+ \mathbf{A}\,\{yz + (\mathbf{B.C})\} + y\mathbf{A.C} + i\, y(\mathbf{A}\times\mathbf{C})$

$+ (\mathbf{A.B})z + i\, z(\mathbf{A}\times\mathbf{B}) + i\mathbf{A}.\,(\mathbf{B}\times\mathbf{C})$

$+\{-\mathbf{A}\times(\mathbf{B}\times\mathbf{C})\}$

[using $(\mathbf{A.B})\mathbf{C} - \mathbf{B}(\mathbf{A.C}) = -\mathbf{A}\times(\mathbf{B}\times\mathbf{C})$]

$= x\{yz + \mathbf{B.C} + y\mathbf{C} + z\mathbf{B} + i\,(\mathbf{B}\times\mathbf{C})\}$

$+ \mathbf{A}\,\{yz + \mathbf{B.C} + y\mathbf{C} + z\mathbf{B} + i\,(\mathbf{B}\times\mathbf{C})\}$

[using $y\mathbf{A.C} + i\, y(\mathbf{A}\times\mathbf{C}) = y\,\mathbf{AC}$, $(\mathbf{A.B})z + i\, z(\mathbf{A}\times\mathbf{B}) = z\,\mathbf{AB}$,

$i\mathbf{A}.\,(\mathbf{B}\times\mathbf{C}) - \mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = i\{\mathbf{A}.\,(\mathbf{B}\times\mathbf{C}) + i\,\mathbf{A}\times(\mathbf{B}\times\mathbf{C})\} = i\,\mathbf{A}(\mathbf{B}\times\mathbf{C})$]

$= (x + \mathbf{A})\,\{yz + \mathbf{B.C} + y\mathbf{C} + z\mathbf{B} + i\,(\mathbf{B}\times\mathbf{C})\}$

$= (x + \mathbf{A})\{(y + \mathbf{B})(z + \mathbf{C})\}$

$= \alpha(\beta\gamma)$

or, $(\alpha\beta)\gamma = \alpha(\beta\gamma)$

Definition 5

Taking x = y = 0 we get from equation (2)

$\mathbf{A}\otimes\mathbf{B} = \mathbf{A.B} + i\mathbf{A}\times\mathbf{B}$ .... (4)

[The symbol ⊗ is chosen for Mixed product.]

This product is called Mixed product of vectors.[6] Mixed product is directly consistent with Pauli matrix algebra and Dirac equation but the quternion product is not directly consistent with Pauli matrix algebra and Dirac equation.[7] Therefore, Mixed product is better than quaternion product.[7]

Definition 6

If $\alpha = x + \mathbf{A}$ and $\beta = y + \mathbf{B}$ then, the adjoint of α, denoted by α*, is defined by

$\alpha^* = x - \mathbf{A}$ .... (5)

Lemma 1

The adjoint in α satisfies

(i) $\alpha^{**} = \alpha$ ....(6)

(ii) $(\alpha + \beta)^* = \alpha^* + \beta^*$ ....(7)

(iii) $(\alpha\,\beta)^* = \beta^*\alpha^*$ .... (8)

Lemma 2

$\text{Scalar}(\alpha) = (\alpha+\alpha^*)/2$ ....(9)

$\text{Vector}(\alpha) = (\alpha-\alpha^*)/2$ ....(10)

Definition 7

The Norm of a Mixed number α is

$N(\alpha) = \alpha\alpha^* = (x + \mathbf{A})(x - \mathbf{A})$

$= (x^2 - \mathbf{A.A} + x\mathbf{A} - x\mathbf{A} - i\mathbf{A}\times\mathbf{A})$

$= (x^2 - A^2)$

Where A is the magnitude of the vector **A**

or, $N(\alpha) = (x^2 - A^2)$ ....(11)

It fulfils the requirement that

$N(\alpha\beta) = N(\alpha)\,N(\beta)$ .... (12)

Definition 8

The inverse of a Mixed number α^{-1} is given by

$\alpha^{-1} = (x - \mathbf{A})/(x^2 - A^2)$ .... (13)

Proof:

$\alpha\,\alpha^{-1} = 1$

or, $\alpha^{-1} = 1/\alpha = 1/(x + \mathbf{A})$

$= (x - \mathbf{A})/\,\{(x + \mathbf{A})\,(x - \mathbf{A})\}$

$= (x - \mathbf{A})/(x^2 - A^2)$

or, $\alpha^{-1} = (x - \mathbf{A})/(x^2 - A^2)$

PROPERTIES OF MIXED NUMBER

Mixed number forms a ring

Mixed number satisfies the following conditions

(i) Additive Associativity:
For all $\alpha, \beta, \gamma \in S$, $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$.

(ii) Additive Commutativity:
For all $\alpha, \beta, \in S$, $(\alpha + \beta) = (\beta + \alpha)$.

(iii) Additive identity:
There exists an element $0 \in S$ such that For all $\alpha \in S$, $0 + \alpha = \alpha + 0$.

(iv) Additive inverse: For every $\alpha \in S$ there exists an element $-\alpha \in S$ such that
$\alpha + (-\alpha) = (-\alpha) + \alpha = 0$.

(v) Multiplicative associativity:
For all $\alpha, \beta, \gamma \in S$, $(\alpha \beta) \gamma = \alpha (\beta \gamma)$.

(vi) Left and right distributivity:
For all $\alpha, \beta, \gamma \in S$, $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$ and
$(\beta + \gamma)\alpha = \beta \alpha + \gamma\alpha$

Therefore, Mixed number forms a ring.

Mixed number forms a division algebra or skew field

Mixed number satisfies the following conditions also

(i) Multiplicative identity: there exists an element $1 \in S$ not equals to zero such that for all $\alpha \in S$, $1\alpha = \alpha 1 = \alpha$

(ii) Multiplicative inverse: For every $\alpha \in S$ not equals to 0, there exists $\alpha^{-1} \in S$ such that
$\alpha \alpha^{-1} = \alpha^{-1}\alpha = 1$.

Therefore, Mixed number forms a division algebra or skew field.

Mixed number does not form a field

The multiplication of Mixed numbers is non-commutative i.e. $\alpha \beta \neq \beta\alpha$.

Therefore, Mixed number does not form a field.

APPLICATIONS OF MIXED NUMBER

Applications of Mixed number in Quantum Mechanics

We could derive[8] the displacement operator, vector differential operator, angular momentum operator and Klein-Gordon equation in terms of Mixed number. Therefore we can conclude that Mixed number is successfully used in Quantum Mechanics.

Displacement operator

The Mixed number displacement operator can be written as

$r_m = (0 + x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$

or, $r_m = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ ....(14)

Using equation (2) and (14) we can write

$(r_m)(r_m) = (0 + x\mathbf{i} + y\mathbf{j} + z\mathbf{k})(0 + x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
$= 0 + (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}).(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
$+ i(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
$= x^2 + y^2 + z^2 = r^2$

or, $(r_m)(r_m) = r^2$ ....(15)

Vector differential operator

The Mixed number vector differential operator can be written as

$\nabla_m = \{0 + \mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y) + \mathbf{k}(\partial/\partial z)\}$

or, $\nabla_m = \mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y) + \mathbf{k}(\partial/\partial z)$ ....(16)

Using equation (2) and (16) we can write

$(\nabla_m)(\nabla_m) = \{0 + \mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y) + \mathbf{k}(\partial/\partial z)\}$
$\{0 + \mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y) + \mathbf{k}(\partial/\partial z)\}$

or, $(\nabla_m)(\nabla_m) = \{\mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y) + \mathbf{k}(\partial/\partial z)\}.$
$\{\mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y) + \mathbf{k}(\partial/\partial z)\}$

$+ i\{\mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y) + \mathbf{k}(\partial/\partial z)\} x \{\mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y)$
$+ \mathbf{k}(\partial/\partial z)\}$

or, $(\nabla_m)(\nabla_m) = (\partial^2/\partial x^2) + (\partial^2/\partial y^2) + (\partial^2/\partial z^2)$

or, $(\nabla_m)(\nabla_m) = \nabla^2$ ....(17)

Where ∇^2 is the Laplacian operator.

Angular Momentum operator

We know that the angular momentum operator of conventional quantum mechanics is

$L_x = i\{z(\partial/\partial y) - y(\partial/\partial z)\}$

$L_y = i\{x(\partial/\partial z) - z(\partial/\partial x)\}$ ....(18)

$L_z = i\{y(\partial/\partial x) - x(\partial/\partial y)\}$

[Considering $\hbar = 1$]

In order to discuss the angular momentum operator in terms of Mixed number we should modify the equation (18) such as

$L_x = \mathbf{i}\{z(\partial/\partial y) - y(\partial/\partial z)\}$

$L_y = \mathbf{j}\{x(\partial/\partial z) - z(\partial/\partial x)\}$ ....(19)

$L_z = \mathbf{k}\{y(\partial/\partial x) - x(\partial/\partial y)\}$

Where $L = L_x + L_y + L_z$ ....(20)

The commutation relations of the angular momentum operator of conventional quantum mechanics are [9]

$$\left.\begin{aligned}[L_x, L_y] &= iL_z\\ [L_y, L_z] &= iL_x\\ [L_z, L_x] &= iL_y\end{aligned}\right\} \qquad \text{................................(21)}$$

[Considering $\hbar = 1$]

Using equation (19) the commutation relations of the angular momentum operators in terms of Mixed number can be written as

$$\left.\begin{aligned}(L_x L_y + L_y L_x) &= iL_z\\ (L_y L_z + L_z L_y) &= iL_x\\ (L_z L_x + L_x L_z) &= iL_y\end{aligned}\right\} \qquad \text{..............................(22)}$$

[Considering $\hbar = 1$]

Proof:

$(L_x L_y + L_y L_x) = [\mathbf{i}\{z(\partial/\partial y) - y(\partial/\partial z)\}][\mathbf{j}\{x(\partial/\partial z) - z(\partial/\partial x)\}] + \mathbf{j}\{x(\partial/\partial z) - z(\partial/\partial x)\}][\mathbf{i}\{z(\partial/\partial y) - y(\partial/\partial z)\}]$

Using the properties of the unique Mixed numbers we get

$(L_x L_y + L_y L_x) = [\mathbf{ik}\{z(\partial/\partial y) - y(\partial/\partial z)\}\{x(\partial/\partial z) - z(\partial/\partial x)\} - \mathbf{ik}\{x(\partial/\partial z) - z(\partial/\partial x)\}\{z(\partial/\partial y) - y(\partial/\partial z)\}]$

or, $(L_x L_y + L_y L_x) = \mathbf{ik}\,[\{z(\partial/\partial y) - y(\partial/\partial z)\}\{x(\partial/\partial z) - z(\partial/\partial x)\} - \{x(\partial/\partial z) - z(\partial/\partial x)\}\{z(\partial/\partial y) - y(\partial/\partial z)\}]$

or, $(L_x L_y + L_y L_x) = \mathbf{ik}\,[\{z(\partial/\partial y)\}\{x(\partial/\partial z)\} -\{z(\partial/\partial y)z(\partial/\partial x)\} - \{y(\partial/\partial z)\}\{x(\partial/\partial z)\} + \{y(\partial/\partial z)\}\{z(\partial/\partial x)\} - \{x(\partial/\partial z)\}\{z(\partial/\partial y)\} +\{x(\partial/\partial z)\}\{y(\partial/\partial z)\} + \{z(\partial\ \partial x)\}\{z(\partial/\partial y)\} - \{z(\partial/\partial x)\}\{y(\partial/\partial z)\}]$

or, $(L_x L_y + L_y L_x) = \mathbf{ik}\,\{-xy(\partial^2/\partial z^2) + y(\partial/\partial x) - x(\partial/\partial y) + xy(\partial^2/\partial z^2)\}$

or, $(L_x L_y + L_y L_x) = \mathbf{ik}\{y(\partial/\partial x) - x(\partial/\partial y)\} = iL_z$

[Using $L_z = \mathbf{k}\{y(\partial/\partial x) - x(\partial/\partial y)\}$]

or, $(L_x L_y + L_y L_x) = iL_z$

Similarly we can show that

$(L_y L_z + L_z L_y) = iL_x$

$(L_z L_x + L_x L_z) = iL_y$

Klein-Gordon Equation

The Klein-Gordon equation of conventional quantum mechanics is[9]

$\{\nabla^2 - (1/c^2)(\partial^2/\partial t^2)\}\Psi - (m^2c^2/\ ^2)\,\Psi = 0$ ....(23)

In terms of Mixed number equation (23) can be written as

$(x + \mathbf{A})(x + \mathbf{B})\,\Psi = (z + \mathbf{C})\,\Psi$ .... (24)

Where $x = (1/c)(\partial/\partial t)$, $\mathbf{A} = \nabla$, $\mathbf{B} = -\nabla$, $z = -(m^2c^2)/\hbar^2$, $\mathbf{C} = 0$

Proof:

According to mixed product equation (24) can be written as

$(x^2 + \mathbf{A.B} + x\mathbf{B} + x\mathbf{A} + i\,\mathbf{AxB})\,\Psi = (z + \mathbf{C})$ ......(25)

Putting the value of x, **A**, **B**, z and **C** in equation (25) we get

$\{ (1/c^2)(\partial^2/\partial t^2) - \nabla^2 - (1/c)(\partial/\partial t)\,\nabla + (1/c)(\partial/\partial t)\,\nabla - i\nabla x \nabla\}\Psi = -\{(m^2c^2)/\hbar^2\}\,\Psi$

or, $\{ (1/c^2)(\partial^2/\partial t^2) - \nabla^2 \}\Psi = -\{(m^2c^2)/\hbar^2\}\,\Psi$

or, $\{ \nabla^2 - (1/c^2)(\partial^2/\partial t^2) \}\Psi \; -\{(m^2c^2)/\hbar^2\}\,\Psi = 0$

Which is exactly same as equation (23)

Applications of Mixed number in Electrodynamics

Mixed number could be applied in dealing with differential operators. Maxwell's equations, The charge conservation law and The Lorentz force are clearly expressed by Mixed number.[10] The electric and magnetic field in terms of scalar and vector potentials are also expressed by Mixed number.[10] Therefore we can conclude that Mixed number is successfully used in Electrodynamics.

In dealing with differential operators

In region of space where there is no charge or current, Maxwell's equation can be written as

(i) $\nabla.\mathbf{E} = 0$ (ii) $\nabla\times\mathbf{E} = -(\partial\mathbf{B})/(\partial t)$ ....(26)

(iii) $\nabla.\mathbf{B} = 0$ (iv) $\nabla\times\mathbf{B} = \mu_0\varepsilon_0(\partial\mathbf{E})/(\partial t)$

From these equations it can be written as [11]

$\nabla^2\mathbf{E} = \mu_0\varepsilon_0(\partial^2\mathbf{E})/(\partial t^2)$ .... (27)

$\nabla^2\mathbf{B} = \mu_0\varepsilon_0(\partial^2\mathbf{B})/(\partial t^2)$

Using equation (5) and (26) we can write

$\nabla \otimes \mathbf{E} = \nabla.\mathbf{E} + i\nabla \times \mathbf{E}$

$= 0 + \{- i(\partial\mathbf{B})/(\partial t)\}$

or, $\nabla \otimes \mathbf{E} = - i(\partial\mathbf{B})/(\partial t)$ .... (28)

or, $\nabla \otimes(\nabla \otimes \mathbf{E}) = \nabla \otimes \{- i(\partial\mathbf{B})/(\partial t)\}$

$= - i(\partial/\partial t)\,\{\nabla \otimes \mathbf{B}\}$

$= - i(\partial/\partial t)\,\{\nabla.\mathbf{B} + i\nabla \times \mathbf{B}\}$

$= - i(\partial/\partial t)\{ 0 + i\,\mu_0\varepsilon_0(\partial\mathbf{E})/(\partial t)\}$

or, $\nabla\otimes(\nabla\otimes \mathbf{E}) = \mu_0\varepsilon_0(\partial^2\mathbf{E}) / (\partial t^2)$..................... (29)

It can be shown that $\nabla \otimes(\nabla \otimes \mathbf{E}) = \nabla^2\mathbf{E}$.......... (30)

From equation (29) and (30) we can write

$\nabla^2\mathbf{E} = \mu_0\varepsilon_0(\partial^2\mathbf{E})/(\partial t^2)$

which is exactly same as shown in equation (27)

Similarly using mixed product it can also be shown that

$\nabla^2\mathbf{B} = \mu_0\varepsilon_0(\partial^2\mathbf{B})/(\partial t^2)$

Therefore mixed product can be used successfully in dealing with differential operators.

The Maxwell's equations.

Let us consider (0 + i**E**), (0 + **B**) and ($\partial/\partial t + \nabla$) are mixed numbers, where **E** is the electric field, **B** is the magnetic field, **∇** is the vector differential operator and i is an imaginary number. Using equation (2), Grassman even product[12] and Grassman odd product,[12] Maxwell's equations can be written as[10]

$\text{even}(\partial/\partial t + \nabla, 0 + \mathbf{B}) - \text{odd}(\partial/\partial t + \nabla, o + i\mathbf{E}) = (0 + 0)$ .... (31)

$\text{even}(\partial/\partial t + \nabla, 0 - \mathbf{E}) - \text{odd}(\partial/\partial t + \nabla, o + i\mathbf{B}) = 4\pi(-\rho + \mathbf{J})$ .... (32)

where, $\text{even}(\alpha,\beta) = \dfrac{\alpha\beta + \beta\alpha}{2}$

and $\text{odd}(\alpha,\beta) = \dfrac{\alpha\beta - \beta\alpha}{2}$

The Conservation Laws

The Mathematical expression of conservation of charge is

$\nabla.\mathbf{J} = -\partial\rho/\partial t$ .... (33)

In terms of Mixed number the equation (33) can be written as[10]

$$\text{scalar}(\partial/\partial t + \nabla)(\nabla.\mathbf{E} + \nabla \times \mathbf{B} - \partial\mathbf{E}/\partial t) = \text{scalar}(\partial/\partial t + \nabla)\{4\pi(\rho + \mathbf{J})\} \quad \text{.......} (34)$$

The Lorentz Force

Mathematically Lorentz Force law[13] can be written as

$$\mathbf{F}=q(\mathbf{E}+\mathbf{v}x\mathbf{B}) \quad \text{.......} (35)$$

In terms of Mixed number equation (35) can be written as[10]

$$\text{even}(q + q\mathbf{v}, 0 + \mathbf{E}) - \text{odd}(q + q\mathbf{v}, 0 + i\mathbf{B}) = (b + \mathbf{F}) \quad \text{.......} (36)$$

The Electric and Magnetic field in terms of scalar and vector potentials

In terms of scalar and vector potentials the Electric and Magnetic field can be written as

$$\mathbf{E}=-\nabla\varphi-\partial\mathbf{A}/\partial t \quad \text{.......} (37)$$

$$\mathbf{B}=\nabla x\mathbf{A} \quad \text{.......} (38)$$

Using the formula of mixed product, Grassman even product and Grassman odd product[12] equation (37) and (38) can be written as[10]

$$E = \text{vector}[\,\text{even}\,(\partial/\partial t-\nabla,\varphi-\mathbf{A})] \quad \text{.......} (39)$$

$$\mathbf{B}=\text{odd}(\partial/\partial t-\nabla,\varphi+i\mathbf{A}) \quad \text{.......} (40)$$

Applications of Mixed number in Special Relativity

Using Mixed number algebra we had found a new most general Lorentz Transformation which, we called Mixed Number Lorentz Transformation.[14] The Lorentz sum of velocities for the Mixed Number Lorentz Transformation is associative, the space generated by the Mixed Number Lorentz Transformation satisfies isotropic property, and Lorentz sum of velocities for the Mixed Number Lorentz Transformation has group property without rotation. The Lorentz sum of velocities for the most general Lorentz Transformation is non-associative, the space generated by it does not satisfy isotropic property and it does not satisfy group property without rotation. Therefore, it is a very innovative application of Mixed number algebra.

CONCLUSION

The mathematical tools of Mixed number algebra i.e. the addition, product, adjoint, norm etc. are clearly explained. We have here applied the Mixed number in Quantum Mechanics, Electrodynamics and special relativity. The complete mathematical tools are necessary for the application of Mixed number. Therefore, this paper will be helpful for the further development and application of Mixed number algebra.

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